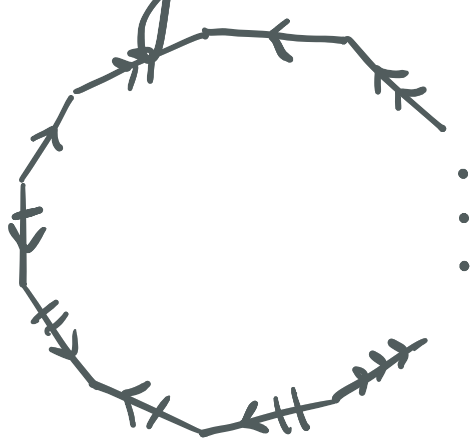
I. Intro to surfaces:

1. Use the fact that every surface of genus g can be obtained from a polygon w/ $4g$ sides and alternating sides identified



to prove that $\chi(S_g) = 2-2g$.

Note that given a polygonal decomposition of a surface with F faces, E edges, and V vertices

$$\chi(S_g) = V - E + F.$$

2) Geometric Intersection #:

a) Compute the geometric intersection number between two nonseparating curves, between two separating curves, and between a separating curve and a nonseparating curve. (Choose pairs of curves that intersect nontrivially!)

b) Suppose α is a separating curve. Prove that $i(\alpha, \beta) \in 2\mathbb{Z}$ for any s.c.c. β.

3) True or False?
"Any two s.c.c. that intersect exactly once are in minimal position."

II. Intro to the curve complex:

1) Prove the curve complex is quasi-isometric to its 1-skeleton

2) Prove that δ-hyperbolicity is a quasi-isometry invariant.

Hint: Quasi-geodesic stability! Every quasi-geodesic is bounded distance from a geodesic in a δ-hyperbolic space.

Def: A quasi-geodesic is a geodesic up to additive and multiplicative constants. For example, the image of a geodesic under a quasi-isometry is a quasi-geodesic.

3) Prove that the curve complex is connected for any closed surface with genus at least 2.

4) Show $\hat{N}(S_{g,n})$ is connected.

5) Are $Map(S_{1,0})$ and $Map(S_{1,1})$ finitely generated? If so, what are their respective finite generating sets? Can you prove/justify your answers?

III. Intro to MCG's

1) The center $Z(G)$ of a group, G, is the collection of elements that commute with every element in the group:

$$Z(G) = \{ g \in G \mid ag = ga \ \forall a \in G \}$$

In this question you will compute the center of Map(S) for S finite type.

Some needed ingredrients:

Thm (Alexander Method)

If $f \in \mathrm{Map}(S)$ is nontrivial, then $\exists$ a s.c.c. c on S s.t. $f(c) \neq c$.

This was extended to infinite type surfaces by
(Hernández Hernández)-Morales-Valdez

Two facts about Dehn twists:

Let $f \in \mathrm{Map}(S)$ and let a, b be s.c.c.

1) $T_a = T_b \iff a = b$
2) $f T_a f^{-1} = T_{f(a)}$

Exercise within an exercise: prove these facts!

Equipped with these ingredients, compute the center of Map(S)!

Does your proof rely on S being finite type? Why or why not?

IV. Intro to big MCG's

1) Given an infinite-type surface S we can define its curve graph, $\mathcal{C}(S)$, in the same way as we do for a finite-type surface.

a) Is $\mathcal{C}(S)$ connected?

b) Infinite diameter?

c) δ-hyperbolic?

2) Check out the survey of Aramayona-Vlamis and the open problem list compiled by Chandran-Patel-Vlamis. Pick out two or three problems

you are interested in and discuss them with at least one other person. Some discussion prompts if you're nervous:

- Share what about these problems piqued your interest.
- What types of tools/topics would you want to learn if you were tackling this problem?
- Is this close (mathematically) to other things you've studied or brand new?

Sketch of finite generation proof:

So what can we show about $\mathrm{Map}(S)$ using the action on $\mathcal{C}(S)$?

Generating $\mathrm{Map}(S)$:

Thm (Dehn-Lickorish)
$\mathrm{Map}(S_g)$ is generated by finitely many Dehn twists about non-separating curves.

Key Lemma: G a group & X a connected 1-dim simplicial complex such that:
- $G \curvearrowright X$ by simplicial automorphisms
- $G \curvearrowright V(X)$ transitively
- $G \curvearrowright E(X)$ transitively

Let $(v,w) \in E(X)$ and choose $h \in G$

so that $h(w) = v$. Then $G = \langle h, \mathrm{Stab}(v) \rangle$.

<u>Key Complex:</u> $\hat{N}(S)$ - modified non-sep. curve complex

vertices - non-separating curves

edges - correspond to non-sep. curves intersecting exactly once

<u>Exercise:</u> Show $\hat{N}(S_{g,n})$ is connected.

<u>Key assumption:</u> $\mathrm{PMap}(S_{0,n})$ is finitely generated by Dehn twists for $n \geq 0$.

<u>Key tool:</u> The Birman exact sequence:

$$1 \longrightarrow \pi_1(S_{g,n}) \longrightarrow \mathrm{PMap}(S_{g,n+1}) \longrightarrow \mathrm{PMap}(S_{g,n}) \longrightarrow 1.$$

Sketch of Proof: Proof by induction

Base case: $\mathrm{Map}(S_{1,0})$ and $\mathrm{Map}(S_{1,1})$ are generated by Dehn twists about the meridian and longitudinal curves. (Exercise)

Inductive step: (On # of punctures)

Let $g \geq 1$ and $n \geq 0$. Note that $\pi_1(S_{g,n})$ is f.g. by non-sep loops. Each of these loops α gives us $P_\alpha = T_a T_b^{-1} \in \mathrm{PMap}(S_{g,n+1})$. By assumption, $\mathrm{PMap}(S_{g,n})$ is f.g. by Dehn twists about non-sep. curves. Lift this generating set to $\mathrm{PMap}(S_{g,n+1})$. So $\mathrm{PMap}(S_{g,n+1})$ is f.g. by Dehn twists about non-sep curves.

Together w/ base case we have completed the proof for $\mathrm{PMap}(S_{1,n})$ for $n \geq 0$.

Inductive step: (On genus)

Since $\hat{N}(S_g)$ is connected and $\mathrm{Map}(S_g)$ acts transitively on ordered pairs of non-sep curves w/ geom. intersection number 1 (by change of coordinates). So apply the key lemma to $\mathrm{Map}(S_g) \curvearrowright \hat{N}(S_g)$.

Let $(v, w) \in E(\hat{N}(S_g))$.

Exercise: Show $T_w T_v(w) = v$.

By the key lemma, $\mathrm{Map}(S_g)$ is generated by $T_w T_v$ and $\mathrm{Stab}(v)$.

All that remains is showing $\mathrm{Stab}(v) = \mathrm{Map}(S_g, a)$ is f.g. by Dehn twists about non-sep. curves.

Exercise: Use the following SES's + the inductive hypothesis to complete the proof.

$$1 \to \underbrace{\mathrm{Map}(S_g, \vec{a})} \to \mathrm{Map}(S_g, a) \to \mathbb{Z}/2\mathbb{Z} \to 1$$

preserve orientation of $\vec{a}$

and

$$1 \to \langle T_a \rangle \to \mathrm{Map}(S_g, \vec{a}) \to \mathrm{PMap}(S_g - a) \to 1$$

$\blacksquare$