Applied Mathematics Seminar
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Positivity Preserving Numerical Schemes for Lubrication Type Equations

by

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Abstract

Lubrication equations such as $h_t + \nabla\cdot(f(h)\nabla\Delta h) = 0$ describe thin films or liquid layers driven by surface tension. Recent studies of singularities, describing rupture of the fluid layer, show that such equations exhibit complex dynamics which can be difficult to simulate accurately. In particular, one must ensure that the numerical approximation of the interface does not show a false premature rupture. Generic finite difference schemes have the potential to manifest such instabilities especially when underresolved. We present new numerical methods, in one and two space dimensions, that preserve positivity of the solution, regardless of the spatial resolution, whenever the PDE has such a property. We also show that the schemes can preserve positivity even when the PDE itself is only known to be nonnegativity preserving. We prove stability and convergence of these methods. We extend this technique to an abstract finite element framework and show, via concrete examples, how this leads to the design of various positivity preserving schemes.