# Applied Mathematics Seminar
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# Deformation of extreme composites

*by*


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## Abstract


The question of how much freedom is to be incorporated in an elasticity theory must ultimately be decided by experiment. However, during the development of the theory of elasticity, it was by no means obvious how much freedom was necessary to describe materials. For example, the early uniconstant theory of Navier is based upon the assumption that forces act along the lines joining pairs of atoms and are proportional to changes in distance between them. This theory entails a Poisson's ratio of 1/4, for all materials. Navier, Cauchy, Poisson, and Lam supported this theory. Experimental measurements (about a century ago) of Poisson's ratio of about 1/3 in common materials led to the replacement of uniconstant elasticity by the more general classical elasticity, following the continuum view of Green, which allows Poisson's ratios between -1 and 1/2.

Recently, cellular solids have been developed which exhibit a controlled negative Poisson's ratio as small as -0.8. Deformation mechanisms in these materials include relative rotation of micro-elements, and non-affine micro-deformation. Cellular solids with a negative Poisson's ratio exhibit superior resilience and toughness as a result of the unfolding of the cells. Studies of these materials conducted via holographic interferometry disclose non-affine deformation.

Additional freedom is possible in solids: the idea of a couple stress can be traced to Voigt in the late 1800's during the formative period of the theory of elasticity, and it was developed further by the Cosserats in 1909. Many theoretical studies were conducted, beginning in the 1960's. In Cosserat elasticity there are characteristic lengths as additional engineering elastic constants. There are a total of six independent elastic constants in an isotropic Cosserat solid. Recent experimental work discloses a variety of cellular and fibrous materials to exhibit such freedom, and the characteristic lengths have been measured. In selected isotropic cellular solids all six of the Cosserat elastic constants have been measured. Several of these constants have been verified by further experiments in geometries different from those used in the original measurements. Holographic studies show that strain can spill over into regions which are classically forbidden, specifically the corners of a square cross-section prism in torsion. Chiral materials are not invariant to inversions: there is a distinction between right and left handed material. Properties such as piezoelectricity, represented by tensors of odd rank, can only occur in chiral materials. Chiral elastic effects cannot be expressed within classical elasticity since the modulus tensor, which is fourth rank, is unchanged under an inversion. Chiral effects can appear in Cosserat solids.

We consider composite material micro-structures, which give rise to high stiffness combined with high viscoelastic loss. We demonstrate that such properties are most easily achieved if the stiff phase is as stiff as possible. We have characterized several candidate materials isothermally over 11 decades of time and frequency with a novel instrument. The rationale is as follows. For some materials, particularly some amorphous polymers, it is possible to infer material properties over a wider range from test results taken at different temperatures. Materials for which such a procedure is possible are called thermorheologically simple. However many materials, particularly composites and materials in which multiple viscoelastic mechanisms are active, are not thermorheologically simple. Direct measurement of properties over many decades is required for a full characterization of the material.