Applied Mathematics Seminar
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Stability for Zhu's combustion problem: The evolution of plane curves by a nonlocal curvature flow.

by

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Abstract

Jingyi Zhu considered a chemical reaction problem between two fluids trapped in the narrow gap between parallel vertical plates, a Hele-Shaw cell. He assumes that the fluids are miscible (no surface tension), have equal viscosity but different density, and that they interact along an interface that can be described by a plane curve. The normal velocity for the curve is given by a curvature dependent combustion term plus a nonlocal term coming from pressure (potential flow given by Darcy's law.) The resulting integro-differential equation is a nonlocal first order perturbation of curvature flow. Zhu found numerically that when the combustion effects are weak, small perturbations of a circular combustion front are unstable and develop into fingers. However, he observed that when combustion effects were strong, perturbations didn't evolve fingers and conjectured that in this case, the circular fronts are stable with respect to small perturbations. Zhu also offered a dispersion analysis for linear fronts as further evidence.

We give analytic results confirming Zhu's conjecture. We show that the perturbed circle combustion front initially tends to grow fingers if the combustion effects are weak enough, and estimate their number. We replace the dispersion analysis by an analysis of the solutions of the linearized equation along the circle, which turns out to be a heat equation on the circle with lower order term involving the Hilbert transform. The linearized equation has solutions which exist for all time and are analytic, but grow at a slower rate than the circle solution. For polynomial initial data, the linearized fingers tend to coalesce into a single finger. Without curvature dependence, the linearized equation has only short time existence but small perturbations may blow up in arbitrarily short time. For the nonlinear problem, the shape of the circle solution is stable under small perturbations of the initial data in the regime where combustion effects are important compared to fluid effects. This means that we can prove long time existence for small perturbations in this regime, and show that the ratio of the perturbed to the unperturbed solutions tends to one.