# UNIVERSITY OF UTAH

# MATH CONTEST



# PSET 3  SPRING 2024

## Problem 1: Fibonacci

The Fibonacci sequence $\{F_n\}_{n\geq 0}$ is defined by the recursion relation

$$F_{n+1} = F_n + F_{n-1}, \quad F_0 = 0, \ F_1 = 1$$

**(a).** Show that
$$F_{n+m} = F_{m+1}F_n + F_m F_{n-1}$$

**(b).** Prove that for any given integer $k$, there exists a Fibonacci number $F_n$ divisible by $k$.

## Problem 2: Bobonacci

Define a **Bobonacci sequence** by the recursion relation

$$B_{n+1} = aB_n + cB_{n-1}$$

with $a, c \in \mathbb{Z}$ and initial condition $B_0 = 0, B_1 = 1$.

Show that, for any given integer $k$, if $\gcd(c, k) = 1$ then one can find a Bobonacci number $B_n$ divisible by $k$.

## Problem 3: The Cube Problem

Margaret Atmosphere is standing at the center of a cube with side length 2. She looks around herself and sees each face of the cube with equal probability 1/6.

Margaret then walks directly towards one face of the cube. She walks a distance of $d < 1$. She now looks in a uniformly random direction. In terms of $d$, what are the new probabilities of her looking at each face?

Submit your answer in integral form. Then evaluate this integral numerically for $d = 1/2$ and submit those probabilities.