

Project Assignment 1, Math 5765, due Tuesday, Feb 14

1. The GARCH(1,1) model for the volatility estimates is given by the equation

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

where $u_n = \log(S_n/S_{n-1})$ is the log return of the stock at time t_n . The parameter $\omega = \gamma V_L$ provides the information about the long-run variance rate as γ is determined through

$$\alpha + \beta + \gamma = 1$$

The model parameters are chosen so that the maximum likelihood

$$\sum_{i=1}^m \left[-\log v_i - \frac{u_i^2}{v_i} \right]$$

is maximized. Note that here $v_i = \sigma_i^2$ are the parameters that determine the distribution of the volatility, and they are in turn determined by parameters ω, α and β .

Download the S&P 500 daily prices for the past year, and try to generate the GARCH(1,1) time series for the volatility by using MLE to determine the parameters ω, α and β . Once a time series for σ_n is generated, you can estimate the volatility of the volatility by performing another GARCH-like procedure on the returns of the volatility ($\log(\sigma_n/\sigma_{n-1})$). Compare this time series with the VIX index data for the past year. What conclusion can you draw from this comparison?

2. To study the option market as a whole, it is useful to analyze the data set for daily closes of the index VIX. Gather the daily returns from the past year and plot them in an appropriate histogram. Use the Metropolis-Hastings algorithm based on the histogram to simulate the index for the next six months to generate a distribution of the VIX index in six months.