

Homework Assignment 7, Math 5765, due April 10 at 5 p.m.

1. This simulation exercise uses different volatility processes to generate stock paths, so that we can gain some insight into the roles played by the volatility in stock behavior and derivative pricing. We begin by assuming that the stock price follows

$$\frac{dS(t)}{S(t)} - \alpha dt = \sigma(t) dW(t)$$

where $W(t)$ is a Brownian motion, α is the expected growth rate and it is assumed to be a known constant, and the volatility $\sigma(t)$ will be specified via another model. We will simulate the stock path using the time horizon $T = 1$, number of business days in a year $N = 250$, and expected return rate $\alpha = 5\%$.

- (a) First we use our basic assumption that the volatility is a constant, and use values 0.25 and 0.5 for σ .
- (b) Next we assume that σ is driven by a simple jump process, to be described as follows. The process $\sigma(t)$ is piecewise constant in t , with jumps of finite size at some random times. The number of jumps $N(t)$ before time t is a Poisson process with intensity λ and it is independent of $W(t)$:

$$P[N(t) = k] = e^{-\lambda t} \frac{(\lambda t)^k}{k!}, \quad k = 0, 1, 2, \dots$$

We assume that $\sigma(0) = 0.5$ and whenever there is a jump, σ goes to a new value that is uniformly distributed between 0 and 1. There can be two approaches to simulate the jump times. One is to directly simulate a sequence of independent exponential rv's, and the other is to simulate whether there is a jump in the next time interval $[t, t + \Delta t]$, as the probability of one jump is approximately $\lambda \Delta t$ for small Δt . For this exercise we use two possible λ values 2 and 10.

- (c) Finally we use a mean reversion process for $V(t) = \sigma^2(t)$, where

$$dV(t) = \kappa(\theta - V(t)) dt + \xi \sqrt{V(t)} dZ(t).$$

Here $Z(t)$ is another Brownian motion that may be correlated with $W(t)$ with correlation coefficient ρ . We assume the following parameters: $V(0) = \sigma^2(0) = 0.25$, $\theta = 0.1$, $\xi = 0.25$, and $\kappa = 1.3$. Use two possible correlation values $\rho = 0$ and -0.75 .

Simulate the stock paths for these models and plot the daily returns in time, along with the volatility plots. Comment on any features you notice that can be attributed to the volatility model.

2. With all the models above, we will try to use Monte Carlo simulations to price European call options on stocks following each of the processes. Let us assume that all the stocks begin with $S(0) = \$50$, the expirations of the calls are all at $T = 0.5$, and the strikes are \$45, \$50 and \$55. The risk-free interest rate is taken to be 1%. Once we get a call price, we should use the Black-Scholes formula to obtain the corresponding implied volatility, and plot them against three different strikes to see what kind of skew/smile for each of the models. It is recommended that at least ten thousand paths should be used for each of the options.