

## Homework Assignment 4, Math 5765, Due March 6

In this project, we will simulate the replicating scheme to hedge a call option contract you supposedly sold to someone. Either a Matlab program or an Excel spreadsheet will suffice to demonstrate the dynamic hedging at work. The following is a description of the procedure.

1. The stock price starts at  $S(0)$ , and we assume that  $S(t)$  follows a geometric Brownian motion process, with an assumed growth rate  $\alpha$  and volatility  $\sigma$ . It is assumed that you just sold an European call option contract (100 shares) with expiration  $T$  and strike  $K$ .
2. You can use the Black-Scholes formula to price the call option and assume that  $V_0 = V(S(0), 0)$  is the price at which you received initially for each share. This means that you have  $X(0) = 100V_0$  to invest, with the goal of replicating the option.
3. At any time  $t \geq 0$ , assume that the value of your hedging portfolio is  $X(t)$ , we will use the information available at this time to determine the new composition of the portfolio and readjust at each time step. You use a portion of the amount to buy certain shares of the stock (assuming that fractions of a share can be traded), determined by the hedge ratio  $\frac{\partial V}{\partial S}$  obtained from the Black-Scholes formula. The rest of  $X(t)$  is deposited in a money market account, with the riskless rate  $r$ .
4. After  $\Delta t$ , the stock price moves to  $S_{t+\Delta t}$ , simulated according to the geometric Brownian motion model. The value of your stock shares in your portfolio changes accordingly. Your money market account grows according to the interest rate  $r$ . The sum is the updated value of your portfolio  $X(t + \Delta t)$ .
5. Repeat steps (3) -(4) until you reach the expiration  $T$ . Compare the value of your replicating portfolio (stock holdings plus the money market account) with the payoff of the call options, and record the difference (the hedging or replication error).
6. Repeat this experiment  $M$  times with different stock price path realizations. Obtain a sample mean and sample variance for these random errors as a measure of how effective this replication strategy is.

Here are some suggested parameters:  $T = 0.25$ ,  $\alpha = 5\%$ ,  $r = 1\%$ ,  $\sigma = 25\%$ ,  $S(0) = \$50$ ,  $K = \$50$ , and  $M = 200$ . Try different values of  $\Delta t$ , such as 0.05 and 0.005 to see the difference in replication errors. With a smaller  $\Delta t$  the replication error is expected to be reduced but it comes with a cost, as there will be transaction costs in reality and more frequent tradings certainly lead to a high transaction cost.