Solutions to Sample Questions

1.

$$\sigma = \sqrt{\frac{(u-1-r\Delta t)(1+r\Delta t-d)}{\Delta t}}$$

Here r is the annualized risk-free interest rate and Δt is measured in years. Since this is only for one period, using a generic r to denote the interest rate over the period is OK, which will leave just r, instead of $r\Delta t$ in the numerator.

2. If we use the approximation $e^x \approx 1+x$ for small x, we get $u = 1.1$ and $d = 0.9$ (note that $u \cdot d$ is no longer 1 here). So the risk-neutral probabilities $\tilde{p} = \tilde{q} = 1/2$. The stock price at the end of period two can take $S(HH) = 60.5$, $S(HT) = S(TH) = 49.5$, and $S(TT) = 40.5$. The payoff of the call is

$$V(HH) = 8.5,\ V(HT) = V(TT) = 0.$$

Following the backward iteration steps, we have $V_0 = 2.125$. If you use the original formula $u = e^{0.1} \approx 1.1052$, the arithmetic would be a little more work but you should end up with just a slightly different answer.

3. In this case $u = 1.2$, $d = 0.8$, and $\tilde{p}, \tilde{q}$ stay at $1/2$, so $S(HH) = 72$, $S(HT) = S(TH) = 48$, and $S(TT) = 32$. The payoff of the put is

$$V(HH) = 0, V(HT) = V(TH) = 4, V(TT) = 20.$$

The put price at time 0 is $V_0 = 7$.

4. Let V denote the price of the contract of one share. We have

$$V(HH) = 8.5,\ V(HT) = V(TH) = -2.5,\ V(TT) = -11.5.$$

So $V_0 = -2$. If you enter this contract, you should get paid in an amount of $2 \times 100 = \$200$. To justify this answer, we consider the replicating portfolio where we borrow \$52 from a bank to buy one share of the stock at the price \$50, so we pocket \$2 upfront. By the time $T = 0.5$, the payoff of this portfolio is $S_T - K$, which is exactly the same as what you would get from this forward contract. Therefore you should get paid \$200 to enter this contract for 100 shares.