Introduction to Mathematical Finance

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Subject of the course

- Modern finance developed using modern and powerful mathematical tools

What is finance?

- borrowing and lending money
- facilitate the capital flow and promote new and efficient economy

Traditional Finance Topics

- personal finance
- corporate finance
- government finance
- international finance

A Prominent topic for the last 50 years

- financial markets:
- how do they operate
- how can they be more efficient
- traditional assets/instruments: cash, equity, bond, etc.
- new instruments: derivatives

Financial Derivatives

- stock options
- commodity futures
- FX futures/options
- interest rate swaps
- interest rate caps/floors
- credit default swaps (CDS)
- collateralized debt obligation (CDO)

Why derivatives?

- the sheer size:
  - size of world stock market at the beginning of 2008: $36.6 trillion
  - total world derivative market: $791 trillion (face or nominal value)
  - 11 times the size of the entire world economy
  - world bond market: $82.2 trillion
- Provide jobs to quantitative oriented professionals (math, physics, CS, etc)

Market Trading

- exchange-traded markets
  - standardized instruments
- over-the-counter markets
  - more exotic, structured instruments

Our Objectives

- Understand various financial derivatives and their relation with the underlying assets
- Look under the hood to find out how they are priced
- Develop criteria in spotting arbitrage opportunities
- Appreciate the ideas behind financial engineering: designing different products to suit customers with different investing objectives
- quantify the risk characteristics within each portfolio

New Buzz Words

- exchange traded funds (ETF)
- high-frequency trading
- pair trading
- algorithmic trading
- statistical arbitrage
- volatility trading

Course Outline

- Introduction to financial derivatives
- definitions and examples
- intentions, popularity with traders, etc.
- pricing based on the price of the underlying
- first-hand experience with data

Binomial models

- one-period models: simplicity
- multi-period models: practical enough that can be used in reality, but still simple to manage
- easy to program
- more general ideas contained and can be extended to continuous time models

No-Arbitrage Principle

- fundamental principle in derivative pricing
- driven by financial arguments
- mathematically self-contained and beautifully framed

Black-Scholes-Merton Model

- Generalization of the binomial model to continuous time
- Stochastic calculus is the key
- price as the solution of the BSM PDE
- Benchmark tool, widely used in trading

American Options

- can be exercised any time before expiration
- more flexible, but at the same time, agonizing to the investors
- more relevant as more options are of this type
- challenging mathematical problem - free-boundary problem is extremely important in many other fields and applications

Random Walks and Brownian Motion

- fundamental probability topic
- Brownian motion present in almost every aspect in life
- rich literature in mathematics
- provide good starting point in modeling

First passage and barrier options

- Another important probability topic with famous results
- barrier options quite useful
- pricing can be very sensitive

Bond mathematics

- Affect our everyday life
- not just one rate, there are so many
- starting from the concept of compounding
- every penny matters
- much more complicated mathematics
- huge size of instruments based on various interest rates