Solutions for Assignment No. 2

Problem 1.7

When the bank takes a long position in this option, it assumes certain risk in the stock, quantified by the numbers of shares of stock required to hedge the option. To eliminate this risk, the bank should take the opposite positions in the stock, and use money market accounts (borrowing from other banks) to finance these stock positions. In particular, the bank should take positions in the stock with number of shares at different times depending on the outcomes

$$\Delta_0 = -0.1733, \ \Delta_1(H) = -0.0667, \ \Delta_1(T) = 0.4667,$$

$$\Delta_2(HH) = 0.3333, \ \Delta_2(HT) = 1, \ \Delta_2(TH) = 0.3333, \ \Delta_2(TT) = 1$$

To execute, it may need to borrow or lend money, and adjust the money market account accordingly. For example, at time 0, it sells 0.1733 shares of the stock and deposits the proceed $4 \times 0.1733 = 0.6933$ in another bank. If the first toss turns head, the required position is short 0.0667 shares, which means that the bank needs to buy 0.1733 - 0.0667 = 0.1066 shares, which would cost 0.8528. The bank's money market account balance is now $0.6933 \times 1.25 - 0.8528 = 0.0138$.

Problem 1.8

1. The goal is to obtain the derivative price at time n as a function v_n of stock price s and price sum y at that time, expressed in terms of the function v_{n+1} , evaluated at appropriate values related to s and y:

$$v_n(s,y) = \frac{1}{1+r} \left(\tilde{p}v_{n+1}(2s,y+2s) + \tilde{q}v_{n+1}(\frac{1}{2}s,y+\frac{1}{2}s) \right).$$

- 2. $v_0(4,4) = 1.216$.
- 3. The delta for hedging is

$$\delta_n(s,y) = \frac{v_{n+1}(2s,y+2s) - v_{n+1}(\frac{1}{2}s,y+\frac{1}{2}s)}{2s - \frac{1}{2}s}$$
$$= \frac{2}{3s} \left(v_{n+1}(2s,y+2s) - v_{n+1}(\frac{1}{2}s,y+\frac{1}{2}s) \right)$$

Problem 1.9

1. The price of a derivative at time zero can be obtained via the following backward induction procedure (here we suppress the explicit dependence on $\omega_1\omega_2\ldots\omega_n$ in cases where no confusion would occur):

Assign v_N for all scenarios $\omega_1 \omega_2 \dots \omega_N$ for n = N - 1 to 0, step -1 for all paths $\omega_1 \omega_2 \dots \omega_n$ leading up to time n

$$\tilde{p}_n(\omega_1\omega_2\dots\omega_n) = \frac{1+r_n-d_n}{u_n-d_n}, \quad \tilde{q}_n = 1 - \tilde{p}_n,$$
$$v_n(\omega_1\omega_2\dots\omega_n) = \frac{1}{1+r_n} \left(\tilde{p}_n v_{n+1}(\omega_1\omega_2\dots\omega_n H) + \tilde{q}_n v_{n+1}(\omega_1\omega_2\dots\omega_n T) \right)$$

 \mathbf{end}

 \mathbf{end}

2. The number of stock shares (delta) to be held in the hedging portfolio is

$$\Delta_n(\omega_1\omega_2\ldots\omega_n) = \frac{v_{n+1}(\omega_1\omega_2\ldots\omega_nH) - v_{n+1}(\omega_1\omega_2\ldots\omega_nT)}{S_n(u_n - d_n)}.$$

3. $V_0 = 9.375.$