Some Relevant Notations and Explanations

1. Zero Rates $r_0(T)$ and $r_{0,n}$

   The zero rate of a T-year zero-coupon bond $r_0(T)$, sometimes also called spot rate, is the rate of interest earned on the bond. It is always quoted with a particular maturity associated with the bond. The subscript 0 refers to the time of observation that is at $t = 0$ (now). Suppose a 5-year zero-coupon bond of face value \$100 that will mature in 5 years is trading at \$90. The implied interest rate $r_0(5)$ (assuming continuous compounding to simplify calculations) is determined from

$$90 \exp(5 r_0(5)) = 100$$

   or

$$r_0(5) = \frac{1}{5} \log\left(\frac{100}{90}\right) \approx 2.11\%.$$

   Prices for different maturity zero-coupon bonds are observed on the market and the above conversion would generate a set of points that can be used to construct a yield curve $r_0(T)$, for example by linear interpolation. In our problem, we pretend that the observed set of zero rates are evaluated from a hypothetical function $r_0(T)$ as shown in the problem.

   With the notations from the exercise, we have

$$r_{0,20} = r_0(T_{20}) = 2.11\%, \qquad B_{0,20} = \$90.$$

2. Forward Rates $f_{0,n}$

   In the text, the forward rate is defined through (6.3.3)

$$F_{n,m} = \frac{B_{n,m}}{B_{n,m+1}} - 1.$$

   It describes the outlook of the future short interest rate observed at time t_n. The problem with the definition is that a unit length time period is assumed from t_n to t_{n+1}, which is not usually used in practice. Typically practitioners use $\Delta t = 0.25$ or $1/12$ for each time period (every quarter or every month). To bring the rates in line with what is usually quoted, we modify the above definition by introducing a time length Δt so

$$f_{n,m} = \frac{F_{n,m}}{\Delta t}.$$

   This is simply the implied interest rate (annualized, or per annum) from time t_m to t_{m+1} , observed at time t_n.

   Note that for the exercise, we use n in the place of m, and $f_{0,n}$ can be inferred from the current yield curve by working on $B_{0,n}, n = 0, 1, \ldots, N$, which in turn can be calculated from $r_{0,n}$.