Homework Assignment 9, Math 5760/6890, due Dec. 9 at 5 p.m.

1. This is an exercise to understand alternative, but equivalent ways to represent the information contained in a yield curve. Let us imagine a yield curve expressed through the zero rate

$$r_0(T) = 0.005 + \alpha T + \beta T^2, \quad 0 \leq T \leq 30$$

   where we pick parameters $\alpha = 0.002$ and $\beta = -0.00003$. We pretend that data points for r_0 are only available at T_n. In another word, only $r_{0,n} = r_0(T_n)$ evaluated from the above function, are given to us for $n = 0, 1, \ldots, N$, with $N = 120$ and $\Delta t = 0.25$.

   (a) Use the data set $\{r_{0,n}, n = 0, \ldots, N\}$ (not the original r_0 function) to obtain zero-coupon bond prices $B_{0,n}$, for $n = 0, \ldots, N$;

   (b) Obtain the forward rates $f_{0,n}$, for $n = 0, \ldots, N-1$;

   (c) Next we compute the price of a zero-coupon bond that matures in 4 years and 7 months. We can obtain an approximation based on the values of $B_{0,18}$ and $B_{0,19}$ by linear interpolation, or we can approximate r_0 for $T = 55/12$, based on $r_{0,18}$ and $r_{0,19}$, and then use the relation between the zero-coupon bond price and the zero rate to obtain an approximation for the bond price. Compare your approximations with the exact price $\exp\left(-r_0(\frac{55}{12})\frac{55}{12}\right)$. Repeat your approximations for another maturity 10 years and 2 months.

2. Suppose a coupon bearing bond with face value \$100, coupons C, paid semiannually, can be priced today as follows.

$$P_0 = \sum_{i=1}^{N} \frac{C}{2} B_0(T_i) + 100 B_0(T_N),$$

   where $B_0(T)$ is the current price of a zero-coupon bond that matures at T, and $T_i, i = 1, \ldots, N$ are the coupon cash flow dates. The yield of the bond (y) is defined through

$$P_0 = \sum_{i=1}^{N} \frac{C}{2} e^{-yT_i} + 100 e^{-yT_N}.$$

   Compute the yield of a 10-year, 5% coupon bond based on the yield curve in Problem 1.

3. Problem 6.3 from the textbook.