Summary of Chapter 2

This chapter illustrates the fundamental arguments behind derivative pricing: the no-arbitrage (or arbitrage-free) principle.

1 Fundamental questions in derivative trading

Once a financial instrument is created and put on the market, the natural question is what price it should be bought/sold. Of course there is the supply and demand mechanism at work, but the guiding principle should reflect its connections with the underlying instruments. A mathematical model would list all possible scenarios for the underlying price and their respective probabilities of occurring. An obvious approach for the price would be estimating the expected value of the payoff based on all scenarios. Unfortunately counter examples are ample in which such prices can lead to arbitrage opportunities.
There is an even more important issue in derivative trading: how to hedge your investments in these financial instruments? This is another reason that the expected value approach would be short of expectations, as we need more than just a price.

2 No-arbitrage principle

From market point of view, a correct price would be one such that arbitrage opportunities will not exist. This is a rather broad statement and we use two mathematical frameworks to build the model to make this statement more specific to follow in developing pricing methodologies.

1. One step binomial model

   The key here is that we have **two** related risky instruments (the stock and an option on it) and **two** possible scenarios, which makes it possible to form a linear combination of two assets (to establish a portfolio) such that the the sum turns out the same price in both scenarios. This is the idea of delta trading and it sounds very artificial and oversimplified, but it turns out to be a reasonable approximation in quite a few cases, enough to be refined to serve as a prototype model for the *complete market* theory. The other side of the coin is that you can form a linear combination of one risky asset (the stock or the option) and a bond to generate a portfolio that turns out the same payoff of the other risky asset, in **both** scenarios!

2. Arbitrage

   In laymen's terms, an arbitrage can be described as a free lunch. We actually need to define arbitrages in rigorous mathematical terms. The crucial idea behind is that no investment will be **guaranteed** a higher return than the riskless interest rate. It is just too good to be true. If you want to make more than the bank deposite, you will have to take some risks: there is a chance that you would not achieve that goal and lose some money.

3 Some direct consequences of no-arbitrage principle

The no-arbitrage principle is so powerful that you can immediately find some applications. In this chapter's discussions, the application is about finding rational upper/lower bounds, without using a specific model for the future prices.

4 Time value of money

A dollar received today is preferred to a dollar received tomorrow. For this reason, investors will never let cash sleep and there is always transactions of borrowing/loaning. In our discussions, the act of borrowing is described as short selling a bond, and loaning out money is equivalent to buying a bond.

5 Assumptions of mathematical finance

These assumptions vary from quite reasonable to obviously idealized.

- Your buying/selling would have no impact on the market;
- You can always find what you want to buy *in any quantity*, at the observed price (liquidity);
- You can sell assets that you do not own (shorting);
- You can buy/sell half (any fraction) of a share of the stock;
- There is no transaction costs.