Math 5760 Project, due Nov. 15 at 5 p.m.

1. Implement the multi-period binomial model to price European calls and puts, and American puts, using the following choices for parameters:

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}},$$

$$p = \frac{1}{2}\left(1 + \left(\frac{r}{\sigma} - \frac{\sigma}{2}\right)\sqrt{\Delta t}\right)$$

   The payoff function $F(S)$ should be an input so the final payoff of the derivative can be calculated as $V_N(S(T)) = F(S(T))$ once the underlying stock price $S(T)$ at time $T = t_N$ is revealed. Your program should take the following inputs: current stock price $S(0)$, strike K, expiration T, volatility σ, risk-free interest rate r, number of periods N, a flag to denote if it is a call or put, and an indication of whether it is European or American if the option is a put.

2. Download the daily prices for the index S&P 500 (SPX) and the ETF ProShare Ultra S&P 500 (SSO) from finance.yahoo.com, for the period beginning 1/3/2011 till now.

   (a) Compute the daily percentage returns and log returns based on close prices for both, and compare to see if SSO daily returns really double the daily returns of SPX as claimed. Also compare the year-to-date returns to see if SSO really doubles the SPX return.

   (b) Use the formulas in the text that include both daily open and close prices to estimate the time dependent historical volatility $\tilde{\sigma}_k$ for day k, based on information from M previous trading days. Choosing $M = 10, 20$, and 30 business days of data, for all available days indexed by k, and plot the estimates as a time series. What can you say about the correlation between historical volatility and SPX itself?

3. The ETF SPY seeks to track the performance of S&P 500 index by holding all of the S&P 500 index stocks, while the leveraged ETF SSO claims to double the daily returns of S&P 500 index. Each of the ETFs has many call and put options written on it. Use the Black-Scholes formula to estimate the implied volatilities of each underlying for 10 most actively traded options according to trading volume, and plot them as functions of the strike (calls and puts should be plotted separately). Observe the relationship between the implied volatility curves for SPY and SSO, and make some comments.