Math 5210, Third Review of Definitions and Theorems

In addition to the topics for the first two reviews, you should be familiar with Chapter 3 and 4 of Royden, and may want to look at the treatment of the same topics in chapter 11 of Rudin. (References: Ro m.n = Royden Chap m, Sec n; Ru m.n = Rudin Chap m, number n; N = Notes)

Definitions

1. Sigma algebra (called sigma-ring in Rudin) (Ru 11.1, Ro 1.4).
2. Borel sets (Ru 11.11 c, Ro 2.7.)
3. Measure (Ro 3.1), outer measure (Ro 3.2), measurable sets, Lebesgue measure (Ro 3.3).
4. Measurable functions, characteristic functions, simple functions (Ro 3.5).
5. Lebesgue integral: for bounded function on set of finite measure (Ro 4.2), of a non-negative function (Ro 4.3), of general function (Ro 4.4).
6. Integrable function (Ro 4.3,4.4), L^1, L^2 functions, L^1, L^2 spaces (N §7).

Theorems

1. Intervals are measurable, sets of measure zero are measurable (Ro 3.3)
2. Characterization of measurable sets (Ro Proposition 3.15), limit properties of measure (Ro Proposition 3.14).
3. Existence of non-measurable sets (Ro 3.4)
4. Properties of measurable functions (Ro 3.5), Littlewood's three principles (Ro 3.6)
5. Upper and lower sums with step functions, simple functions, relation between Riemann integrability and Lebesgue integrability for bounded functions on sets of finite measure (Ro 4.2).
6. Convergence Theorems: Bounded Convergence (Ro 4.2), Monotone Convergence (Ro 4,3), Dominated Convergence (= Lebesgue Convergence) (Ro 4.4). Know when to apply them, and examples that show that the hypotheses are needed.
7. Fatou's Lemma (Ro 4.3). Know examples that show inequality can be strict.
8. L^p spaces are complete (for $p = 1, 2$) (Ro 6.3).