1. The purpose of this exercise is to prove, directly from the least upper bound property of real numbers, that Cauchy sequences of real numbers converge.

   Suppose that $\{x_n\}$ is a Cauchy sequence of real numbers.

   (a) Prove that the set $\{x_n : n \in \mathbb{N}\}$ is bounded.

   (b) For each $N \in \mathbb{N}$, let $E_N = \{x_n : n \geq N\}$. Observe that it follows from (a) that

      i. For each $N \in \mathbb{N}$, E_N is a bounded set,

      ii. The sequence E_N is nested: $E_N \supset E_{N+1} \supset E_{N+2} \ldots$

      Let $a_N = \inf(E_N)$ and $b_N = \sup(E_N)$. Prove that for all $M, N \in \mathbb{N}$, $a_M \leq b_N$.

   (c) Let $A = \{a_N : N \in \mathbb{N}\}$ and $B = \{b_N : N \in \mathbb{N}$. Prove that $\sup(A) = \inf(B)$. Call their common value x_0. Prove that $x_0 = \lim\{x_n\}$.

2. Problem 8 of Rudin, Chapter 1 (p 22). Justify your answer from the definition of ordrered field.

3. Problem 9 of Rudin, Chaper 1. Explain why the first part does not contradict the last problem.