1. Let $\{x_n\}$ be a Cauchy sequence of rational numbers, and let $[\{x_n\}] \in \mathbb{R}$ be the real number it represents. Prove that $\{x_n\}$ converges to the real number $[\{x_n\}]$.

   Note: To prove this you must use the definitions from class, which you can find in the notes posted at *http://www.math.utah.edu/˜toledo/5210Notes.pdf* . First you need to use the inclusion of $\mathbb{Q}$ in $\mathbb{R}$ as constant sequences, as in (1) of section 2.2 of the notes. This means that you need to convert $\{x_n\}$ to the double sequence

   $$x_{i,n} = x_n \quad \text{(constant in } i\text{)}.$$

   Then apply definition (2.5) of limit to this double sequence and the limit sequence $\{x_i\}$.

2. Let X be any set and let $B(X)$ denote the set of *bounded* real valued functions on X:

   $$B(X) = \{f : X \to \mathbb{R} \text{ such that } f(X) \subset \mathbb{R} \text{ is a bounded set.}\},$$

   Define a distance function on $B(X)$ by

   $$d_\infty(f,g) = \sup_{x \in X}\{|f(x) - g(x)|\}.$$

   Prove that $(B(X), d_\infty)$ is a metric space. (The only difficulty is in proving the triangle inequality).

3. Prove that the limit of a uniformly convergent sequence of bounded functions is bounded (Rudin, Chapter 7, Problem 1).

4. Prove that $(B(X), d_\infty)$ is a complete metric space. (You can follow step by step the proof given in class that $(C(X), d_\infty)$ is a complete metric space.)

5. (Part of Rudin, Chapter 7, Problem 4): For which subsets of $[0, \infty)$ does the series

   $$\Sigma_{n=1}^{\infty} \frac{1}{1 + n^2 x}$$

   converge uniformly?