1. Write down a complete proof of the following statement, following the sketch in class. Be careful to write all the steps in proper logical order:

   Suppose that $\{x_n\}$ is a Cauchy sequence of rational numbers, and suppose that it is not the case that $\lim x_n = 0$. Then there is a fixed number $\epsilon_0 > 0$ and a fixed natural number N_0 so that $|x_n| \geq \epsilon_0$ for all $n \geq N_0$.

   (Note: this was used to define $1/[\{x_n\}]$ for $[\{x_n\}] \neq [0]$. The same type of argument can be used to prove the following statement, that is used in checking the order relation on $\mathbb{R}$:

   Suppose that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences of rational numbers. Exactly one of the following is true:

   (a) $\lim(x_n - y_n) = 0$.

   (b) There exists a natural number N such that $x_n < y_n$ for all $n \geq N$,

   (c) There exists a natural number N such that $x_n > y_n$ for all $n \geq N$.)

2. Problems 4, 5 of Rudin, Chapter 1 (p 22).

3. Problem 16 of Rudin, Chaper 3 (p 81). Change part (a) to read "Prove that $\{x_n\}$ decreases monotoniclally and $x_n > \sqrt{\alpha}$ ". Then, add to part (c): "Therefore $\lim x_n = \sqrt{\alpha}$. "