

1. Verify the Cauchy-Riemann equations for the following functions.

(a) $z^3 = (x + iy)^3$ (first expand).

(b) $f(z) = z^3 + z^2 + z$.

(c) $\cos(z) = \cos(x) \cosh(y) - i \sin(x) \sinh(y)$.

(d) $\sin(z) = \sin(x) \cosh(y) + i \cos(x) \sinh(y)$.

2. Prove that the following functions are not analytic:

(a) $f(z) = \bar{z}$

(b) $f(z) = |z|$

(c) $f(z) = \Re(z)$

(d) $f(z) = \cos(\bar{z})$

3. Show that if $f(z) = u(x, y) + iv(x, y)$ is analytic, then so is $\overline{f(\bar{z})}$. Then check this directly for $\cos(\bar{z})$.