Math 2210, Review for Midterm 3

The third midterm exam (April 8) will be on sections 12.8 to 13.8 of the textbook. As usual, material is cumulative, you may need material form previous sections in order to answer questions on the newer material. The best way to prepare is to do the homework problems. In this case, the probems on maxima and minima in set 6, and sets 7,8,9,10.

Topics

1. Section 12.8 (Maxima and minima):

   (a) How to find interior maxima and minima of f: These are among the stationary points, where the gradient is zero. Solve the equation $\nabla f(x, y) = 0$ for (x, y) in two variables, or $\nabla f(x, y, z) = 0$ for (x, y, z) in three variables.

   (b) Classify critical points by second derivative test into maxima, minima, saddle points.

   (c) Critical points on the boundary and absolute maxima and minima on a closed bounded region that has both interior and boundary: solve $\nabla f = 0$ in the interior, treat boundary either by parametrizing it or by Lagrange method. Then test the value of f all the points you found to find smallest and largest.

2. Section 12.9 (The method of Lagrange multipliers):

   (a) Find maxima and minima of $f(x, y)$ subject to $g(x, y) = 0$ by solving $\nabla f = \lambda\nabla g, g(x, y) = 0$.

   (b) Same in three variables: maxima and minima of $f(x, y, z)$ subject to $g(x, y, z) = 0$ by solving $\nabla f = \lambda\nabla g, g(x, y, z) = 0$.

   (c) Know how to use this method to find maxima and minima of f on a closed bounded region with boundary $g = 0$: $\nabla f = 0$ at interior points, $\nabla f = \lambda\nabla g$ and $g = 0$ at boundary points.

3. Section 13.1 (Double integrals over rectangles) : Be familiar with the ideas here, but there will be no questions on this section.

4. Section 13.2 (Iterated Integrals): Know how to do integrals

$$\int_a^b \int_c^d f(x, y)dydx, \quad \int_c^d \int_a^b f(x, y)dxdy$$

   over rectangles $a \leq x \leq b$, $c \leq y \leq d$, and what the order of integration means: $dy\ dx$ versus $dx\ dy$.

5. Section 13.3 (Double integrals over non-rectangular regions) This is probably the most important section in Chapter 13.

   (a) Know how to go back and forth between pictures for regions and descriptions by inequalities $a \leq x \leq b$, $\phi_1(x) \leq y \leq \phi_2(x)$ or $c \leq y \leq d$, $\psi_1(y) \leq x \leq \psi_2(y)$.

For example, a triangle with vertices at $(0,0),(1,0),(1,3)$ (draw a picture!) can be described either as

$$0 \le x \le 1,\ 0 \le y \le 3x \quad \text{or} \quad 0 \le y \le 3,\ \frac{y}{3} \le x \le 1.$$

(b) Know how to evaluate integrals with variable limits, and how to change order of integration, for example, if given one of the two following integrals, find the other:

$$\int_0^1 \int_0^{3x} f(x,y)\ dydx \quad = \quad \int_0^3 \int_{\frac{y}{3}}^1 f(x,y)\ dxdy.$$

6. From 13.4 (Double integrals in polar coordinates):

   (a) Know how to find limits of integration in polar coordinates, usually $a \le \theta \le b$, $\phi_1(\theta) \le r \le \phi_2(\theta)$, and evaluate integrals $\int_a^b \int_{\phi_1(\theta)}^{\phi_2(\theta)} f(r,\theta) r dr d\theta$.

   (b) Know how to change between rectangular and polar coordinates, for example

$$\int_0^{1/\sqrt{2}} \int_x^{\sqrt{1-x^2}} f(x,y) dy\ dx \quad = \quad \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 f(r\cos(\theta), r\sin(\theta))\ r dr d\theta.$$

7. Section 13.5 (Applications of double integrals): Be familiar with this section. If I give you a problem here, for example, on the center of mass, I will give you the formulas you need.

8. Section 13.6 (Surface area) Be ready to compute integrals of the form

$$\int\int_R \sqrt{1 + f_x^2 + f_y^2}\ dA$$

   for the area of the graph of $z = f(x,y)$. I will remind you of the formula.

9. Section 13.7 (Triple integrals) Same comments as sections 13.2, 13.3 but one more variable. This time there are six possible orders of integration.

10. Triple integrals in cylindrical and spherical coordinates: Know when to use them and how to change variables. I will give you the formulas for change of variables, but you need to know how to change limits, for example

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} f(x,y,z) dz\ dy\ dx = \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{\sqrt{1-r^2}} f(r\cos\theta, r\sin\theta, z)\ r dz\ dr\ d\theta$$

   which in spherical coordinates becomes

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\ \rho^2 \sin\phi d\rho\ d\theta\ d\phi.$$