

The second midterm exam (March 4) will be on sections 11.1 to 12.7 of the textbook, with more emphasis on sections 11.6 to 12.7 (the material covered since the first midterm). You should be familiar with these sections and have worked the problems in homework sets 1 through part of 6 (with more emphasis on homeworks 3 to 6). The best way to prepare for the midterm is to do all the homework problems.

Topics

1. From section 11.6 (Lines and Tangent Lines in Three– Space).
 - (a) Parametric and symmetric equations of a line: line through a point in a direction, line through two points.
 - (b) Line perpendicular to a plane..
 - (c) Tangent line to a curve at a point on the curve.
2. From 11.2 (Curvature and Components of Acceleration): You're encouraged to learn this material, but I will not ask you any questions on this topic.
3. From 11.8 (Surfaces in Three Space)
 - (a) Know the quadric surfaces and how to distinguish them: ellipsoids, hyperboloids of one or two sheets, paraboloids.
 - (b) Know how to distinguish the above surfaces from “cylinders”
 - (c) Be able to sketch surfaces from their equations.
4. From 11.9 (Cylindrical and Spherical Coordinates):
 - (a) Know how to convert between one of them and rectangular coordinates.
 - (b) Be able to sketch equations in these coordinates, such as $r = \text{constant}$ or $\theta = \text{constant}$ or $\rho = \text{constant}$
5. From 12.1 (Functions of Two or More Variables)
 - (a) Be able to graph functions of two variables
 - (b) Be able to sketch level sets of functions of two or three variables.
6. From 12.2 (Partial Derivatives):
 - (a) Be able to find partial derivatives of functions of any number of variables.
 - (b) Same for higher order partial derivatives.
7. From Section 12.3(Limits and Continuity)
 - (a) Be able to decide if limits of the form $\lim_{(x,y)\rightarrow(0,0)} f(x,y)$ exist or not (for the examples given in this section).

(b) If limits exist, be able to find their value.

8. From Section 12.4 (Differentiability)

(a) Remember that differentiability means having a good linear approximation. If f is differentiable at (x_0, y_0) , the linear approximation gives the tangent plane: if $z = f(x, y)$ and $z_0 = f(x_0, y_0)$, then

$$z = z_0 + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0) + \epsilon \quad (1)$$

where $\epsilon/\sqrt{(x - x_0)^2 + (y - y_0)^2} \rightarrow 0$ as $(x, y) \rightarrow (x_0, y_0)$. The terms before the error term give the linear approximation, and the equation of the tangent plane to $z = f(x, y)$ at the point (x_0, y_0, z_0) .

(b) DEFINITION OF THE GRADIENT:

$$\nabla f(\mathbf{p}) = \frac{\partial f}{\partial x}(\mathbf{p}) \mathbf{i} + \frac{\partial f}{\partial y}(\mathbf{p}) \mathbf{j} = \left\langle \frac{\partial f}{\partial x}(\mathbf{p}), \frac{\partial f}{\partial y}(\mathbf{p}) \right\rangle. \quad (2)$$

This is the definition for functions of two variables. For functions of three variables there would be a third component $\frac{\partial f}{\partial z}(\mathbf{p})$, etc.

(c) Using the definition of the gradient, equation (1) reads

$$f(\mathbf{p} + \mathbf{h}) = f(\mathbf{p}) + \nabla f(\mathbf{p}) \cdot \mathbf{h} + \epsilon, \quad \text{where } \epsilon/||\mathbf{h}|| \rightarrow 0 \text{ as } \mathbf{h} \rightarrow 0. \quad (3)$$

9. From Section 12.5 (Directional Derivatives and Gradients)

(a) Definition of directional derivative of f at \mathbf{p} in direction of the unit vector \mathbf{u} is

$$D_{\mathbf{u}}f(\mathbf{p}) = \lim_{h \rightarrow 0} \frac{f(\mathbf{p} + h\mathbf{u}) - f(\mathbf{p})}{h}. \quad (4)$$

(b) DIRECTIONAL DERIVATIVES IN TERMS OF THE GRADIENT

$$D_{\mathbf{u}}f(\mathbf{p}) = \mathbf{u} \cdot \nabla f(\mathbf{p}) = ||\nabla f(\mathbf{p})|| \cos(\theta), \quad \text{where } \theta = \angle(\mathbf{u}, \nabla f(\mathbf{p})). \quad (5)$$

(c) CONSEQUENCES:

- i. $-||\nabla f(\mathbf{p})|| \leq D_{\mathbf{u}}f(\mathbf{p}) \leq ||\nabla f(\mathbf{p})||$, equality on the right if and only if \mathbf{u} has the direction of $\nabla f(\mathbf{p})$, equality on the left if and only if \mathbf{u} had the direction opposite to that of $\nabla f(\mathbf{p})$.
- ii. An equivalent way of saying this: the direction of the gradient is the direction of greatest increase of f at \mathbf{p} , and the largest rate of increase is the magnitude of the gradient. The direction of fastest decrease is the direction opposite to the gradient, and the rate of decrease in this direction is the negative of the magnitude of the gradient.
- iii. $D_{\mathbf{u}}f(\mathbf{p}) = 0$ if and only if $\mathbf{u} \perp \nabla f(\mathbf{p})$. $\nabla f(\mathbf{p})$ is perpendicular to the level set of f at \mathbf{p} . (Level set = level curve in two variables, level surface in three variables).

10. From Section 12.6 (The Chain Rule)

- (a) Know the various versions of the chain rule, for example, if $z = f(x, y)$ and $x = x(t), y = y(t)$, then $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$
- (b) Know how to use it, for instance, look at example 2 in 12.6.
- (c) Implicit differentiation, for instance, example 6 in 12.6.

11. From Section 12.7 (Tangent Planes and Approximations)

- (a) Know how to find equation of tangent plane and normal line to $F(x, y, z) = 0$ at $\mathbf{p} = (x_0, y_0, z_0)$:
 - Normal vector: $\nabla f(\mathbf{p}) = \langle \frac{\partial f}{\partial x}(\mathbf{p}), \frac{\partial f}{\partial y}(\mathbf{p}), \frac{\partial f}{\partial z}(\mathbf{p}) \rangle$
 - Tangent plane: $\frac{\partial f}{\partial x}(\mathbf{p})(x - x_0) + \frac{\partial f}{\partial y}(\mathbf{p})(y - y_0) + \frac{\partial f}{\partial z}(\mathbf{p})(z - z_0) = 0$,
 - Normal line: Parametric equation $\mathbf{p} + t\nabla f(\mathbf{p})$
- (b) Special case: Tangent plane to $z = f(x, y)$ at (x_0, y_0, z_0) given by (1).
- (c) Use for approximation: $\Delta z \cong \frac{\partial f}{\partial x}(\mathbf{p})\Delta x + \frac{\partial f}{\partial y}(\mathbf{p})\Delta y$ See examples 3 and 4 of section 12.7.

Some Formulas I'll give you with the midterm

1. Line through $\mathbf{p} = (x_0, y_0, z_0)$ in the direction of $\mathbf{v} = \langle a, b, c \rangle$:

(a) *Parametric Equation* : $\mathbf{p} + t\mathbf{v} = (x_0 + at, y_0 + bt, z_0 + ct)$

(b) *Symmetric Equations*: $(x - x_0)/a = (y - y_0)/b = (z - z_0)/c$.

2. Gradient $\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

3. Directional derivative at \mathbf{p} in direction \mathbf{u} : $D_{\mathbf{u}}f(\mathbf{p}) = \mathbf{u} \cdot \nabla f(\mathbf{p})$ which can also be written as $D_{\mathbf{u}}f(\mathbf{p}) = \|\nabla f(\mathbf{p})\| \cos(\angle(\mathbf{u}, \nabla f(\mathbf{p})))$.

4. Tangent plane to $z = f(x, y)$ at (x_0, y_0, z_0) given by

$$z - z_0 = \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$$

5. Normal vector to $F(x, y, z) = 0$ at $\mathbf{p} = (x_0, y_0, z_0)$ is $\nabla F(\mathbf{p})$.