

The first midterm exam (February 4) will be on sections 11.1 to 11.5 of the textbook. You should be familiar with these sections and have worked the problems in homework sets 1 and 2. The best way to prepare for the midterm is to do all the homework problems.

Topics

1. From section 11.1 (Cartesian Coordinates in Space).
 - (a) How to use coordinates.
 - (b) Distance between two points in space, equation of a sphere with given center and radius.
 - (c) Sketch some simple graphs, for example, cylinders.
 - (d) Find the length of a curve $L = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} dt$ in simple cases.
2. From 11.2 (Vectors):
 - (a) Vectors: know how to draw pictures and how to use components.
 - (b) Operations with vectors: addition, multiplication by scalars.
 - (c) Know magnitude $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$, unit vector in direction of \mathbf{v} is $\frac{\mathbf{v}}{\|\mathbf{v}\|}$.
 - (d) Applications to simple physics and geometry problems, such as Example 3, p.562 or the homework problems in set 1.
3. From 11.3 (Dot Product):

- (a) KNOW THE TWO FORMULAS FOR THE DOT PRODUCT:

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta.$$

- (b) Know how to find angles between vectors: $\theta = \arccos\left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}\right)$ and how to apply this to find other angles, for example, angles of a triangle .
- (c) \mathbf{u} and \mathbf{v} are perpendicular (= orthogonal) if and only if $\mathbf{u} \cdot \mathbf{v} = 0$.
- (d) PROJECTION: See Figures 6, 7 on p. 569 and explanations on bottom of p 569 to top of 570: the *vector projection* or simply *projection* of \mathbf{u} on \mathbf{v} :

$$\text{Projection of } \mathbf{u} \text{ on } \mathbf{v} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\right)\mathbf{v}$$

and the *scalar projection* is $\|\mathbf{u}\| \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|}$.

- (e) Know how to apply the above to find physical quantities like work = $\mathbf{F} \cdot \mathbf{D}$.
- (f) EQUATION OF A PLANE: The plane perpendicular to $\langle A, B, C \rangle$ containing the point (x_0, y_0, z_0) is $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ or $Ax + By + Cz = D$ where $D = Ax_0 + By_0 + Cz_0$. This means that the projection of (x, y, z) on $\langle A, B, C \rangle$ is the same as the projection of (x_0, y_0, z_0) on $\langle A, B, C \rangle$.

- (g) Know how to find the distance from a point to a plane, or the distance between two parallel planes, either from the formulas in examples 10 and 11 of this section, or by reasoning.
4. From 11.4 (Cross Product)
- (a) Be familiar with the two formulas for the cross-product. Do not expect you to memorize them, but you should know how to interpret them: Figure 1 and Theorem A on p 575, and know the basic properties: Theorems B and C.
 - (b) Know how to use this formula to : find a vector perpendicular to two given vectors, find the equation of a plane through three given points.
 - (c) Know how to use this formula to find the area of a parallelogram and the volume of a parallelepiped.
5. From 11.5 (Vector -valued functions, motion):
- (a) Vector functions $\mathbf{F}(t)$, how to differentiate them and integrate them.
 - (b) If $\mathbf{r}(t)$ is position, know how to find velocity $\mathbf{v}(t) = \mathbf{r}'(t)$, speed $\|\mathbf{v}(t)\| = \frac{ds}{dt}$, acceleration $\mathbf{a}(t) = \mathbf{v}'(t)$.
 - (c) Know how to do the above backwards: starting from $\mathbf{a}(t)$ and initial values of velocity and position, find $\mathbf{r}(t)$ (see examples 6 and 7 p 584).

Some Formulas I'll give you with the midterm

1. *Vector projection* of the vector \mathbf{u} on the vector \mathbf{v} :

$$\text{Projection of } \mathbf{u} \text{ on } \mathbf{v} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \right) \mathbf{v}$$

and the *scalar projection* is $\|\mathbf{u}\| \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|}$.

2. Plane perpendicular to $\langle A, B, C \rangle$ containing the point (x_0, y_0, z_0) is $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
3. Distance from the point (x_1, y_1, z_1) to the plane $Ax + By + Cz = D$ is

$$\frac{|Ax_1 + By_1 + Cz_1 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

4. Cross-Product:

$$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} + (u_3v_1 - u_1v_3)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

$\mathbf{u} \times \mathbf{v}$ is perpendicular to both \mathbf{u} and \mathbf{v} , $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin \theta$

5. Area: $\|\mathbf{u} \times \mathbf{v}\| =$ Area of parallelogram determined by \mathbf{u} and \mathbf{v} .
6. Volume: $|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}| =$ volume of parallelepiped determined by $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

Observe that notably absent from this list are the formulas for the dot product. I expect you to know the two formulas for the dot product.