Math 2210-2 Final Exam - 5/6/09 Name

Justify all your answers and show all your work. Correct answers with no work could get at most half credit.

1. Let C be the curve $x = 3\cos(t)$, $y = 3\sin(t)$, $z = 4t$, $0 \le t \le \pi/2$.
   (a) (5 pts) Find the velocity and acceleration vectors to C.
   (b) (5 pts) Find the length of C.
   (c) (5 pts) Find the value of $\int_C x \, ds$.
   (d) (5 pts) Find the value of $\int_C xdx + ydy + zdz$.

2. (20 pts) Find the maximum and minimum *values* of the function $f(x, y) = 2x^2 + 5y^2$ on the set $\{(x, y) : x^2 + y^2 \le 1\}$.

3. Let $\mathbf{F}(x, y, z) = xy\mathbf{i} + xz^2\mathbf{j} + y^2z\mathbf{k}$.
   (a) (5 pts) Find $div\mathbf{F}$.
   (b) (10 pts) Find $curl\mathbf{F}$
   (c) (5 pts) Is $\mathbf{F}$ the gradient of a function? Explain.

4. Find the value of the following integrals. In each case you need to change variables.
   (a) (10 pts)
   $$\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)^{10} \, dydx$$
   (b) (10 pts)
   $$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2)^{3/2} \, dzdydx$$

5. (10 pts) Draw a picture of the region and find the limits for the triple integral $\int\int\int_R x \, dzdydx$, where R is the region in the first octant under the plane $\frac{x}{2} + y + z = 1$. You don't have to find the numerical value of the integral.

6. (10 pts) Find the value of the line integral
   $$\oint_C (x^3 - y)dx + (x + y^2)dy$$
   where C is the boundary of the rectangle $0 \le x \le 2$, $0 \le y \le 1$, counterclockwise. (Suggestion: avoid doing the line integral directly.)

Math 2210-1 Final Exam - 12/11/12 Name

Justify all your answers and show all your work. Correct answers with no work could get at most half credit.

1. (10 pts) Find the length of the curve $(\cos(3t), \sin(3t), 4t), 0 \leq t \leq 4\pi$.

2. Let $f(x, y) = x^2y + xy^2$, and let $\mathbf{p}$ be the point $(1, 2)$.

   (a) (10 pts) Find the directional derivative of f at $\mathbf{p}$ in the direction $(\mathbf{i} - \mathbf{j})/\sqrt{2}$.

   (b) (5 pts) Is the function increasing or decreasing in that direction? Explain.

   (c) (5 pts) Find the unit vector in the direction of greatest increase of f at the point $\mathbf{p}$.

   (d) (5 pts) Find the rate of increase of f at $\mathbf{p}$ in the direction of greatest increase.

3. (15 pts) Find the maximum and minimum values of $f(x, y) = x - y$ on the set $\{x^2 + y^2 \leq 1\}$.

4. (15 pts) Find the volume enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$. Sketch.

5. Let $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$

   (a) (5 pts) Find $curl(\mathbf{F})$.

   (b) (5 pts) Is $\mathbf{F}$ the gradient of a function? Explain.

6. Let $\mathbf{F}(x, y) = 2xy\mathbf{i} + (x^2 + 2y)\mathbf{j}$.

   (a) (8 pts) Find a function $f(x, y)$ with $\nabla f = \mathbf{F}$.

   (b) (7 pts) Evaluate $\int_C 2xy\ dx + (x^2 + 2y)\ dy$ where C is the curve formed by the line segment form $(0, 0)$ to $(2, 1)$ followed by the segment from $(2, 1)$ to $(5, 2)$.

7. (10 pts) Evaluate $\oint_C (e^{x^2} - 2xy)\ dx + (x^2 + \cos(y))\ dy$ where C is the boundary of the square with vertices $(0, 0), (2, 0), (2, 1), (0, 1)$, counterclockwise.