

1. Length of curve  $(x(t), y(t), z(t)), a \leq t \leq b$  is  $\int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} dt = \int_a^b ds$ .
2.  $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$ .
3.  $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2v_3 - u_3v_2)\mathbf{i} + (u_3v_1 - u_1v_3)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$ .
4.  $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin \theta = \text{area of parallelogram}, (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \text{volume of parallelepiped}$ .
5. Plane  $A(x - x_0) + B(y - y_0) + C(z - z_0) = Ax + By + Cz - D = 0 \perp A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ .
6. Line  $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$  through  $(x_0, y_0, z_0)$  in direction  $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ .
7. Position  $\mathbf{r}(t)$ , velocity  $\mathbf{v}(t) = \frac{d\mathbf{r}}{dt}$ , speed  $\|\mathbf{v}(t)\| = \frac{ds}{dt}$ , acceleration  $\mathbf{a}(t) = \frac{d\mathbf{v}}{dt}$ .
8.  $\nabla f = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$ .
9. Directional derivative at  $\mathbf{p}$  in direction  $\mathbf{u}$ :  $D_{\mathbf{u}}f(\mathbf{p}) = \mathbf{u} \cdot \nabla f(\mathbf{p})$ .
10. Chain rule:  $\frac{d}{dt}(f(\mathbf{r}(t))) = \nabla f(\mathbf{r}(t)) \cdot \frac{d\mathbf{r}}{dt}$ .
11. Tangent plane to  $z = f(x, y)$  at  $(x_0, y_0, z_0)$ :  $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$ .
12. Second derivative test at  $(x_0, y_0)$  where  $\nabla f((x_0, y_0)) = 0$ : If  $D = (f_{xx}f_{yy} - f_{xy}^2)((x_0, y_0))$ , then: (1) :  $D > 0, f_{xx} > 0$  local min; (2)  $D > 0, f_{xx} < 0$  local max; (3) :  $D < 0$  saddle pt.
13. Area of graph  $z = f(x, y)$  over region  $S$  in  $xy$ -plane:  $\int \int_S \sqrt{f_x^2 + f_y^2 + 1} dA$ .
14. Polar coordinates:  $x = r \cos \theta, y = r \sin \theta, dA = r dr d\theta$ .
15. Spherical coordinates:  $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi, dV = \rho^2 \sin \phi d\rho d\theta d\phi$ .
16.  $\text{div}(\mathbf{F}) = \nabla \cdot \mathbf{F} = M_x + N_y + P_z$ .
17.  $\text{curl}(\mathbf{F}) = \nabla \times \mathbf{F} = (P_y - N_z)\mathbf{i} + (M_z - P_x)\mathbf{j} + (N_x - M_y)\mathbf{k}$ .
18.  $\text{curl}(\nabla f) = \mathbf{0}$ .
19. If  $C$  is a curve  $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, a \leq t \leq b$ , then  $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C Mdx + Ndy + Pdz = \int_a^b (M \frac{dx}{dt} + N \frac{dy}{dt} + P \frac{dz}{dt}) dt$  and  $\int_C f ds = \int_a^b f(\mathbf{r}(t)) \|\frac{d\mathbf{r}}{dt}(t)\| dt$ .
20. If  $\mathbf{F}$  is conservative means  $\mathbf{F} = \nabla f$ . If  $\mathbf{F}$  conservative, then  $\text{curl}(\mathbf{F}) = \mathbf{0}$ .
21. If  $C$  is a curve from  $P$  to  $Q$ ,  $\int_C \nabla f \cdot d\mathbf{r} = f(Q) - f(P)$ .
22. Green's Thm:  $\oint_{\partial S} Mdx + Ndy = \int \int_S (N_x - M_y) dA$
23. Divergence Thm in plane:  $\oint_{\partial S} \mathbf{F} \cdot \mathbf{n} ds = \int \int_S \text{div}(\mathbf{F}) dA, \mathbf{n} = \text{outward unit normal}$ .
24. Divergence Thm in space:  $\int \int_{\partial S} \mathbf{F} \cdot \mathbf{n} dS = \int \int \int_S \text{div}(\mathbf{F}) dV, \mathbf{n} = \text{outward unit normal}$ .