Math 1180 - Spring 2006

Practice Exam 2

March 3, 2006

You are allowed to bring your calculator and one page of note (double-sided) for the exam. Textbook and other notes are not allowed.

PROBLEMS

1. After a day of growth, the measurement L describing the length of a group of worms has p.d.f.

$$f(l) = 1.5(l + l^2)$$

   with $0.5 \leq l \leq 1.0$

   (a) Sketch the pdf and show that it satisfies all the necessary requirements.

   (b) Find the probability that a worm is less than 0.75 mm in length.

   (c) What is the probability that a worm is between 0.7 and 0.8 mm long?

   (d) How many worms out of 1000 would you expect to be longer than 0.75 mm?

   (e) Find the expectation of the worm length.

2. A bacterium switches between moving and tumbling/stopping. It switches from moving to tumbling during a given second with probability 0.2 and from tumbling to moving with probability 0.1.

   (a) Write down the information above in condition probability notation. Write a Markov chain model.

   (b) Write down a discrete dynamical system for the probability that the bacterium is moving. Find the equilibrium of the system.

   (c) If the bacterium swim at a velocity of 10 μm/s (when it's at a moving state), find the average speed of the bacterium after a long time.

3. A laboratory is testing for a rare bacterial mutant which appears in 1% of cells. A test correctly identifies a mutant with probability 0.9 and gives a false positive with probability 0.02

   (a) Find the probability of a positive test.

   (b) Find the probability that a cell that tests positive actually has the mutation. Comment on the reliability of this test.

4. Suppose that researchers in the Department of Holticulture are developing data on the last frost. They measure the time t continuously in months after April 1st (for example $t = 0.5$ corresponds to the midnight on April 15). Let T be a random variable representing the time of the last frost. They have found that

$$Pr\{T \leq t\} = 2t - t^2$$

for $0 \leq t \leq 1$

   (a) Check that this c.d.f. makes sense. When is the last possible day for the last frost?

   (b) Find the median date of the last frost.

   (c) Find the probability that plants will be damaged by frost if they are planted ot midnight on April 20?

   (d) Find the probability density function of T.

   (e) Find the mean date of the last frost. Compare it with the median and explain why one is greater than the other?

   (f) When do you think would be the best time to plant and why?

5. Suppose a population obeys

$$N_1 = R_1 N_0$$

where R_1 is a random variable that takes on the value 1.5 with probability 0.6 and the value 0.5 with probability 0.4. Suppose $N_0 = 1$

   (a) Find the expectation and the geometric mean of N_1

   (b) Find its variance and the variance of $ln(N_1)$.

6. Researchers in the Department of Ecology and Evolution are rumored to have discovered a population of tropical birds whose numbers jump between 5 and 10 and take on no other values. Let H represent the event that the population is "high" (value 10) and L represent the event that the population is "low" (value 5). Suppose that $Pr(H_{t+1}|L_t) = 0.3$ and $Pr(L_{t+1}|H_t) = 0.2$.

   (a) Give a complete description of the dynamics of this population. That is, write down the complete Markov chain model of the system.

   (b) Write down a discrete-time dynamical system for the probability that the population is high

   (c) Find the long-term probability.

   (d) The population can double, halve or stay the same. Find the probability that each of these events after a long time and compute the geometric mean growth rate. Does your answer make sense?