Math 1180 - Mathematics for Life Scientists

Computer Assignment 9 - Binomial Distribution

Due Friday, April 14, 2006

Getting Started

Warm up Maple for today's problems with the commands:

```
> with(stats);
> iread(binomial);
```

Binomial Package

The binomial package you just load contains the following commands you may want to use today,

1. Probability Mass Function $b(k, n, p)$

   ```
   > b(k,n,p);
   ```

   This command gives the probability of obtaining *exactly* k successes in n trials each with success probability p.

2. Cumulative Distribution Function $B(k, n, p)$

   ```
   > B(k,n,p);
   ```

   This command gives the the probability of obtaining k *or fewer*successes in n trials each with success probability p.

Problems

1. Suppose that the probability p_t that a molecule inside a cell after t time steps is

   $$p_t = 0.9^t$$

   Input this as a Maple function.
   Suppose 100 molecules start out in a cell. You can define the probability of having *exactly* 50 molecules as a function of t by typing

   ```
   > p50:= t -> b(50,100,p(t));
   ```

   and the probability of having 50 *or fewer* molecules as a function of time

   ```
   > P50:= t -> B(50,100,p(t));
   ```

(a) Plot the following as functions of time t on a single graph with reasonable axes: (a) the probability that 50 molecules are inside at time t, and (b) the probability that 10 molecules are inside at time t. Explain in words the shape of the curve. At what time is it most likely to find exactly 50 molecules inside the cell? What about the most likely time to find exactly 10 molecules inside the cell?

(b) Find out the numerical value of the probability that there are exactly 50 molecules inside the cell at time $t = 12$. What about the probabilities that there are 10 molecules inside at $t = 12$? Which of the two is more likely to occur?

(c) What is the expected number of molecules at time $t = 12$? What is the expected number of molecules as a function of time? Input this as a Maple Function and plot it. Describe the behavior you see in your plot.

(d) Plot these c.d.f.'s as functions of time: (a) the probability of having 50 or fewer molecules inside at time t, and (b) the probability of having 10 or fewer molecules inside the cell at time t. Explain in words the meaning of these plots. Do the cdf's tend to 1? Indicate which one approaches 1 sooner and explain why this makes sense.

(e) Plot the variance as a function of time. Estimate the time, t^*, when the variance takes on its maximum. Why does the variance become small after a long time? (Think about it - how many molecules are left inside after a long time).

(f) Plot the coefficient of variation as a function of time. Use it to deduce the predicatability of the process. Your previous plot should indicate that the variance decreases for larger values of t. Does this mean that the process becomes more predictable after t^*? For large t, the variance is small but the coefficient of variation is large, explain why this makes sense.

You may want to consult the discussion in your text to answer some of the questions above. See page 642-644.

2. Consider a situation where two types of molecules leave a cell with different probabilities during each minute. Suppose the first molecule leaves with probability $q1$ and that the second molecule leaves with probability q_2.
Suppose we begin with n_1 number of molecules of type 1 and n_2 of type 2, so the total number of molecules is $n = n_1 + n_2$. If we did not have a way to differentiate between these two types, we would just find the probability that a molecule will leave at each minute by taking the average,

$$q = q_1 \frac{n_1}{n} + q_2 \frac{n_2}{n}$$

Consider the following cases,

(a) $n_1 = n_2 = 50$, $q_1 = 0.1$, $q_2 = 0.9$
(b) $n_1 = n_2 = 50$, $q_1 = 0.6$, $q_2 = 0.4$
(c) $n_1 = 90$, $n_2 = 10$, $q_1 = 0.1$, $q_2 = 0.9$
(d) $n_1 = 10$, $n_2 = 90$, $q_1 = 0.1$, $q_2 = 0.9$

Find out the average number of molecules of each type that remain inside after 5 minutes,

$$E[N_1] = n_1 * (1 - q_1)^5$$
$$E[N_2] = n_2 * (1 - q_2)^5$$

So the mean of the total molecules still remaining after 5 minutes is $E[N_1] + E[N_2]$. Compare this with the average behavior, i.e. the expected number out of n that would remain inside if molecules leave with an average probability q per minute. How important is it to distinguish among types of molecules?