Math 1180 - Mathematics for Life Scientists

Computer Assignment 8 - Some Problems in Probability

Due **Friday**, April 7, 2006

We will take a break from using Maple this week. Instead do the following thought exercises. You may work in groups. List the name of people you collaborated with in your final report.

Problems

1. A player throws a fair dice and simultaneously flip a coin. Assume that the outcome of the dice and of the coin are independent of each other. If the coin lands head, her score will be twice the value that appears on the dice. If the coin lands tail instead, her score will be one-half of the dice value.

   (a) Find the expected value of the score.

   (b) If this game is repeated for 50 times, what is the expected total score?

   Hint: The score is just a product of two random variables. Use independence to find the expected value of this product.

2. A group of N people throws their hats into the center of a room. The hats are mixed up and each person randomly selects one. Find the expected number of people that select their own hat. To get you started, set up the problem as follows. Let X_i be the outcome for each person,

$$X_i = \begin{cases} 1 & \text{when the } i\text{th person selects his own hat} \\ 0 & \text{otherwise} \end{cases}$$

   Then, the number of matches X can be written as the sum $X = X_1 + X_2 + ... + X_N$.

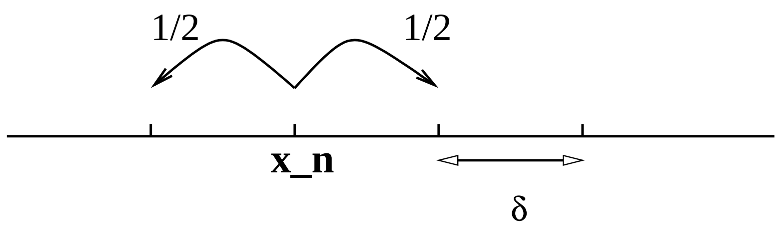
3. *Swimming E. Coli*

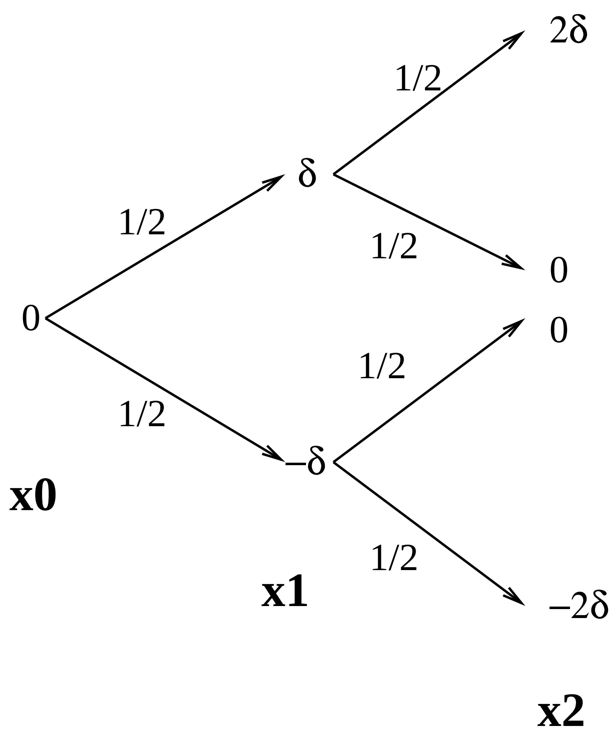
   Consider the following one-dimensional model of a swimming E. Coli. Let x_i denote the position of the bacterium at time t_i. At every time interval τ (i.e. $t_i = i\tau$), the bacteria can step over a distance δ to the right with probability 0.5 and a distance δ to the left with probability 0.5 (drunkard's walk). So, the position of the cell at time t_{n+1} can be written as

$$x_{n+1}) = \begin{cases} x_n + \delta & \text{with probability } 0.5 \\ x_n - \delta & \text{with probability } 0.5 \end{cases}$$

   (a) Find the average positions x_{n+1}. Does this really mean that the bacterium is stuck at the origin and not moving?

   (b) Assume that at time $t = 0$, the bacteria is at the origin, $x_0 = 0$. Find the variance at time t_1, t_2, ..., t_n (See attached diagram).
   What is the standard deviation of x_n?





(c) Remember the assumption that a switch in direction is made every time interval τ, so the actual time t is related with n according to $t = n\tau$. Substitute $n = t/\tau$ to your formula for the standard deviation for x_n. The standard deviation as we know is a measure of spreading from the mean.

(d) Data collected from tracking bacterium movement indicates that δ is approximately 30μm and $\tau = 1$ second. Find out how much the bacterium has spread within 10 seconds, 1 minutes, 1 hour, and 1 day.

(e) It is observed that bacteria can move over 9 cm in one day. Do you consider this random walk approach as a good model to describe bacterial swimming? What do you think should be done to improve the model?