Math 1180 - Mathematics for Life Scientists

Computer Assignment 2

Due Tuesday, January 24, 2005

Getting Started

Log in to your computer, and click the computer icon at the bottom of the screen to open an xterm window. Create a folder called lab1 by typing

```
> mkdir lab2
```

Open Maple by clicking on the Maple icon on the bottom of the screen. Once in Maple, go to File, Save As, and save the worksheet as **lab2.mws** in the **lab2** folder.

Newton's Law of Cooling

Newton's law of cooling states that the rate of change of the internal temperature of a body is proportional to the difference between the internal and external/ambient temperature. In class, we looked at an example where the ambient temperature is constant A. Today, in this lab., we will consider the case when the ambient temperature $A(t)$ is oscillating with period T according to

$$A(t) = 20 + sin\left(\frac{2\pi t}{T}\right) \tag{1}$$

Let $H(t)$ denote the internal body temperature then the differential equation describing it is

$$\frac{dH}{dt} = \alpha(A(t) - H) \tag{2}$$

You will need to solve the differential equation above using Maple. Recall how you used the command **dsolve** to solve the differential equation

$$\frac{dH}{dt} = 2 - H \tag{3}$$

$$H(0) = 0$$

```
> de := {diff(h(t),t)=2-h(t) , h(0) = 0};
> h  := unapply(rhs(dsolve(de,h(t))),t);
```

Problems

1. Input the function $A(t)$ above. Make sure that you type π as Pi with capital P.

   Using an initial condition of $H(0) = 20$, find the solution of equation (2). Maple should be able to do it without setting numerical values for parameters α and T.

2. **Specific Heat Constant**
   Investigate the effect of varying the specific heat by changing the constant α. Materials with high specific heat have low values of α. An example of this type of materials is water; they retain heat for a long time and experience small rates of heat change for a given temperature difference. On the other hand, materials with low specific heat, such as metals, have high values of α. They tend to loose heat rapidly and experience high rates of heat change for a given temperature difference.

   Use values of $\alpha = 0.1$, 1.0, 10.0, 100.0 and set $T = 1$. For each value of α above, graph the functions $H(t)$ and $A(t)$ on the same plot for $0 \leq t \leq 5.0$. For example, to do this for $\alpha = 1.0$, type the following code into Maple

   ```
   > T:= 1.0;
   > alpha := 1.0;
   > plot([H(t),A(t)], t=0..5, color=[blue,red]);
   ```

   Try to explain how the solutions are changing as α is varied. In each case, try to estimate the phase delay, i.e. distance between the peak of $A(t)$ and that of $H(t)$. To see things better, you may need to change the range of t over which you are plotting the curve. How does the amplitude of $H(t)$ change as you vary α? For which value of α does the solution most closely track the ambient temperature? Why?

3. **Steady State Solution**
   The equation that we studied today is non-autonomous or non-homogeneous. The exact solution can be written as a summation of two things $H(t) = H_{tr}(t) + H_p(t)$ where $H_{tr}(t)$ describes the transient part of the solution and $H_p(t)$ gives the steady-state/long-term behavior.

   Change the initial condition to $H(0) = 50$ and $H(0) = 0$. Use $\alpha = 10.0$ and $T = 1.0$. For each case, plot $H(t)$ and $A(t)$ together over a reasonable range of t. On a long term, how does the solution behave? Look at the solution obtained using **dsolve**. Can you identify the transient part and the steady-state part of the solution? If so, label them on your report. Do you think $H_p(t)$ and $A(t)$ should be related? Explain how and why.