Math 1180 - Mathematics for Life Scientists

Computer Assignment 1

Due Tuesday, January 17, 2005

Getting Started

Log in to your computer, and click the computer icon at the bottom of the screen to open an xterm window. Create a folder called lab1 by typing

```
> mkdir lab1
```

Open Maple by clicking on the Maple icon on the bottom of the screen. Once in Maple, go to File, Save As, and save the worksheet as **lab1.mws** in the **lab1** folder.

Pure-time and Autonomous Differential Equations

We will use **dsolve** to compare a **pure-time differential equation** with an **autonomous differential equation**. Our pure time differential equation is

$$\frac{dV}{dt} = 2 - t. \tag{1}$$

Our autonomous equation describes Newton's law of cooling

$$\frac{dH}{dt} = 2 - H, \tag{2}$$

with ambient temperature $A = 2$ and rate of decay $\alpha = 1$. We will solve each equation starting from two initial conditions.

Refer to the last section on your Maple Tutorial to learn the basics of using the **dsolve** command. There are some tricks that need to be done to get Maple to give you a function back after solving the equation. For example, to get Maple to solve the two equations above using the initial conditions $V(0) = 0$ and $H(0) = 0$ respectively, use the following:

```
> diffv0 := {diff(v(t),t)=2-t,v(0)=0};
> V0 := unapply(rhs(dsolve(diffv0,v(t))),t);
> diffh0 := {diff(h(t),t)=2-h(t),h(0)=0};
> H0 := unapply(rhs(dsolve(diffh0,h(t))),t);
```

Now, try to create functions $V4$ and $H4$ by solving the same differential equations with initial conditions $V(0) = 4$ and $H(0) = 4$.

Problems

1. Plot $V0$, $V4$, $dV0/dt$ and $dV4/dt$ as functions of time for $t = 0$ to $t = 4$. Plot the derivatives with commands like `diff(V0(t),t)`. Label the curves and write the corresponding formulas. Where is $V0$ increasing? $V4$?
   Recall that equilibria refer to points where the function is not changing i.e its derivative is exactly zero. Does V have an equilibrium? If so, where is it?

2. Do the same for $H0$ and $H4$.

3. For Newton's law of cooling plot the rate of change of temperature as a function of temperature for $0 \leq H \leq 4$. Now think of the solution $H0$. Points on your graph correspond to different values of t. At $t = 0$, the temperature is 0 and the rate of change is 2, corresponding to the point $(0, 2)$. Mark this point. Find and mark the points corresponding to $t =$1, 2, 3 and 4. Do the same for the solution $H4$. Draw arrows on your graph to indicate which way the temperature is changing.