

Math 1180 - Spring 2006

Homework 1

Due January 18

1. Review of Pure-Time Differential Equations

Section 4.5 – 39, 45

2. Autonomous Differential Equations

Section 5.1 – 2, 4, 6, 8, 15, 18, 25, 29, 38, 39

3. Euler's Method – Comparing Approximate and Exact Solutions

- (a) Use Euler's method to find an approximate solution to the differential equation

$$\frac{dy}{dt} = y$$

with $y(0) = 1$. Use a step-size $\Delta t = 0.25$ and find values up to $t = 1.0$. Compare the value that you get for y_n using Euler's method with the exact solution $y(t) = e^t$. What is the error i.e. the difference between the exact solution and the approximation? Repeat the problem using $\Delta t = 0.1$. How does the error change using a smaller time-step?

- (b) Now use Euler's method to approximate the solution of

$$\frac{dy}{dt} = -10y$$

with $y(0) = 1$. Again, use a step-size $\Delta t = 0.25$ and find values up to $t = 1.0$. Compute the error to compare your result with the exact solution $y(t) = e^{-10t}$. Does your approximate solution behave in a similar way as your exact solution? What happens if you use a smaller step-size, $\Delta t = 0.1$? Is your approximate solution now decaying to zero? What do you think you should learn from this problem?