

# AN APPLICATION OF THE RIEMANN-ROCH FORMULA IN THE BLOW-UP OF A NONSINGULAR AFFINE SCHEME

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Let  $(A, \mathfrak{m})$  be a regular local ring. The Rees algebra  $A[\mathfrak{m}t]$  of  $A$  is the homogeneous ring of the blow-up  $X = \text{Proj } A[\mathfrak{m}t]$  of  $\text{Spec } A$  at  $\text{Spec}(A/\mathfrak{m})$ . Let  $P_M(t)$  be the Hilbert-Samuel polynomial of a finitely generated  $A$ -module  $M$  such that  $P_M(n) = \ell(M/\mathfrak{m}^n M)$  for  $n \gg 0$ . The *Rees module*  $R_{\mathfrak{m}}(M) = \bigoplus_{n \geq 0} \mathfrak{m}^n M$  of  $M$  is a graded module over  $A[\mathfrak{m}t]$ .

Over a graded ring that determines a projective scheme, the Riemann-Roch formula relates the Euler characteristic of a graded module to its Chern characters. Thus if all but the zero-th sheaf cohomology modules vanish, the Euler characteristic becomes the Hilbert function of the graded module. In particular, this gives a precise form for the Riemann-Roch formula for a finite product of projective spaces.

We will discuss an approach of relating the Hilbert-Samuel polynomial  $P_M(t)$  of  $M$  over  $A$  to the Chern characters of the graded Rees module  $R_{\mathfrak{m}}(M)$  over  $A[\mathfrak{m}t]$ . The main aim is to establish a Riemann-Roch formula at the level of the blow-up algebra of a regular local ring  $A$ . We apply the Grothendieck-Riemann-Roch formula on the projective spaces to achieve this goal. This project is initiated by a conversation with Srinivas. It is a joint work with Claudia Miller.