INTRODUCTION TO POLYNOMIAL CALCULUS

PROBLEMS

1. Straight Lines

In problems (1) through (6) find the slope of the line containing the indicated two points.

(1) $(0, 1)$ and $(1, 2)$
(2) $(2, 3)$ and $(4, 7)$
(3) $(1, 1)$ and $(3, 2)$
(4) $(1, 4)$ and $(3, 2)$
(5) $(-2, 3)$ and $(3, 1)$
(6) $(-2, 0)$ and $(0, 2)$

In problems (7) through (12) find the equation of the line with the indicated slope and passing through the indicated point.

(7) slope 2 and point $(0, 0)$
(8) slope 5 and point $(1, 2)$
(9) slope -3 and point $(2, -1)$
(10) slope $\frac{1}{2}$ and point $(1, 1)$
(11) slope $-\frac{2}{3}$ and point $(0, 5)$
(12) slope 7 and point $(-2, 0)$

(13) Find the equation of the line with slope 3 and y-intercept 1
(14) Find the equation of the line with slope $\frac{4}{3}$ and y-intercept 2
(15) Find the slope and y -intercept for the line with equation $6x - 2y = 4$
(16) Find the slope and y -intercept for the line with equation $2x + 5y = 3$
(17) Find the equation of the line which passes through $(1, 1)$ and is parallel to the line $y = 3x + 2$.
(18) Find the equation of the line which passes through $(2, -1)$ and is parallel to the line which passes through $(2, 0)$ and $(3, 2)$.
(19) Find the equation of the line which passes through $(1, 0)$ and is perpendicular to the line $y = 3x + 2$
(20) Find the equation of the line which bisects the line segment from $(0, 0)$ to $(2, 4)$ at a right angle.
(21) Find the equation of the line which passes through $(0, 1)$ and is perpendicular to the line $x = 3$.
(22) Find the equation of the line which passes through $(2, 0)$ and is perpendicular to the line $y = 1$.
(23) If a perpendicular line is drawn from the point $(1, 1)$ to the line $2y - x = 4$, at what point does it meet this line? What is the distance from the point $(1, 1)$ to the line $2y - x = 4$.
(24) What is the distance from the point $(0, 1)$ to the line $y = 2x - 3$?
(25) What is the distance from the line $y = 2x$ to the parallel line $y = 2x + 3$?

2. Slope of a Curve

In problems 1 - 8 , you are to find the slope of the curve $y = f(x)$ at the point where x has the indicated value by calculating $\frac{f(x+h)-f(x)}{h}$ and determining what number it approaches as h approaches 0.

(1) $f(x) = 3x + 2, \quad x = 1$
(2) $f(x) = x^2, \quad x = 0$
(3) $f(x) = x^2, \quad x = 2$
(4) $f(x) = x^2 - 3, \quad x = 1$
(5) $f(x) = x^2 + 2x - 1, \quad x = 0$
(6) $f(x) = 3x^2 - 2, \quad x = 1$
(7) $f(x) = x^3, \quad x = 1$
(8) $f(x) = x^3, \quad x = 0$

In problems 9 - 14 you are to find $f'(x)$ by calculating $\frac{f(x+h)-f(x)}{h}$ and determining what it approaches as h approaches 0.

(9) $f(x) = x$
(10) $f(x) = 2x + 5$
(11) $f(x) = 3x^2$
(12) $f(x) = x^2 - 2x + 3$
(13) $f(x) = x^3$
(14) $f(x) = x^3 + x^2$

3. Derivative of a Polynomial

(1) Find the derivative of x^9
(2) Find the derivative of $2x^{50}$
(3) Find the derivative of $3x - 6$
(4) Find the derivative of $x^3 - 2x + 4$
(5) Find the derivative of $2x^4 + x^3 - 5x^2 + x + 2$
(6) Find the derivative of $x^{11} - 2x^9 + 15x$
(7) Find the slope of the curve $y = x^3$ at the point $(1, 1)$
(8) Find the slope of the curve $y = x^2$ at the point $(0, 0)$
(9) Find the slope of the curve $y = x^3 - x^2$ at the point $(1, 0)$
(10) Find the slope of the curve $y = x^4 - 2x^3 + 5x + 3$ at the point where $x = -1$
(11) Find the slope of the curve $y = 2x^{50} - 50x^2$ at the point where $x = 1$
(12) For what values of x does the curve $y = x^2 - 2x + 3$ have positive slope? Negative slope? Zero slope?
(13) If a ball is thrown straight up in such a way that its height t seconds later is

$$s(t) = -16t^2 + 32t + 6$$

find the velocity of the ball at t seconds after it is thrown. At what time t does the ball reach its maximum height (hint: the velocity will be positive before this time and negative after it). How high does the ball get?

(14) In the previous problem, what is the acceleration of the ball at any time t?

4. Antiderivatives of Polynomials

(1) Find $\int (2x - 3)\, dx$

(2) Find $\int (3x^2 - 4x + 5)\, dx$

(3) Find $\int (x^5 + 2x^3 + 1)\, dx$

(4) Find $\int (10x^9 - 8x)\, dx$

(5) Find the antiderivative of $x^2 - 5$ that has value 2 when $x = 0$.

(6) Find the antiderivative of $8x^3 - 2x$ that has the value 4 when $x = 1$.

(7) Find the antiderivative of $2x^3$ that has the value 1 when $x = 1$.

(8) Find the antiderivative of $x^3 - x$ that has the value 1 when $x = 2$.

(9) If a ball is thrown straight up with initial velocity of 64 ft/sec, what will its velocity be after t seconds? At what time t will it achieve its maximum height?

(10) If the ball in the last problem was thrown from an initial height of 6 feet, what will its height be after t seconds? What is the maximum height it achieves?

5. Definite integrals

1. Find $\int_1^5 (x^2 - 2x + 1)dx$.

2. Find $\int_0^2 (x^3 + 2)dx$.

3. Find $\int_0^1 (x^4 - x^5)dx$.

4. Find $\int_0^1 (x^n - x^{n+1})dx$, for any $n \geq 0$.

5. Find the area under the curve $y = x^2 + 5x$ from $x = 3$ to $x = 4$.

6. Find the definite integral of $y = x^{10} - x^9$ from $x = 1$ to $x = 3$.

7. A particle travels along a horizontal line so that its velocity at time t is $v(t) = 2t + 3t^2 + 1$ feet per second. Suppose that at time $t = 1$ the particle is at the origin. What is the location of the particle at time $t = 3$?