The point of this note is to modify an explanation found in Section 3.6.1 of "Mathematical Physiology".

There it is stated that if the rate at which an ion channel changes from the closed state C to the open state O is α then the closed time probability distribution is exponential,

$$P_C(t) = 1 - \exp(-\alpha t). \tag{1}$$

This is correct but misleading. This is because for the sodium channel model shown in Fig. 3.12, there are two ways to leave the closed state, to the open state at rate α and to the inactivated state at rate δ. This means that the time spent in the closed state is exponentially distributed with rate $\alpha + \delta$,

$$P_C(t) = 1 - \exp(-(\alpha + \delta)t). \tag{2}$$

Now if one examines data of closed times by looking at the length of time a channel is closed between subsequent openings, these times will be distributed exponentially with rate $\alpha + \delta$, not with rate α. This is because even though the transitions that are observed are all to the open state, the time of those transitions are the same as any possible transition out of the closed state. Another way of saying this is that because there are some transitions to the inactivated state, there are possible transitions to the open state that were not observed, since the transition to the inactivated state occurred first. Hence the distribution of opening times is not representative of transitions from closed to open alone, i.e., not at rate α.

The implication of this (useful for exercise 17) is that one can use the closed time data to estimate the parameter $\alpha + \delta$, not α, as suggested by the text.