Determine whether the following statement is True or False.

1. A system of four linear equations in three unknowns is always inconsistent.
2. There exists a $3 \times 4$ matrix with rank 4.
3. If $A$ is a $3 \times 4$ matrix and vector $\vec{v}$ is in $\mathbb{R}^4$, then vector $A\vec{v}$ is in $\mathbb{R}^3$.
4. If the $4 \times 4$ matrix $A$ has rank 4, then any linear system with coefficient matrix $A$ will have a unique solution.

$$\begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$
5. $\text{rank} \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} = 2$.

6. There exists a matrix $A$ such that $A \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 7 \end{pmatrix}$.

7. Vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$.

8. If $A$ is a nonzero matrix of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, then the rank of $A$ must be 2.

9. If $A$ and $B$ are matrices of the same size, then the formula $\text{rk}(A + B) = \text{rk}(A) + \text{rk}(B)$ must hold.

10. If $A$ is a $3 \times 4$ matrix of rank 3, then the system $A\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ must have infinitely many solutions.

\textit{Note:} January 17,