1. Change of bases matrix from $B$ to $U$, where $B$ & $U$ are bases of $V$.

Let $B = (f_1, \ldots, f_n)$ be a basis of $V$.

Then $S_{B \to U} = [f_1]_B [f_2]_B \cdots [f_n]_U$ & $S_{B \to U}(f_i)_B = [f_i]_U$.

2. The $B$-matrix of $T$:

Let $B = (f_i)$ & $T : V \to V$, a linear map.

Then the $B$-matrix of $T$ is $[T(f_1)]_B [T(f_2)]_B \cdots [T(f_n)]_B$.


Let $A$ be the $U$-matrix of $T$, $B$ the $B$-matrix of $T$ and $S = S_{B \to U}$ the change of bases from $B$ to $U$.

Then $AS = SB \Rightarrow B = S^{-1}A$.

Ex. Let $T : P_2 \to P_2$ by $T(f) = f' + tf(0)$. Let $B = (1, t, t^2)$ & $U = (1, t + t^2, t^2)$ be $B$-bases of $P_2$.

Then $S = S_{B \to U} = [1 1 1] [t-1]_u [t+t^2]_u = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.

The $U$-matrix $A = [T(1)]_u [T(t)]_u [T(t^2)]_u = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

So the $B$-matrix $B = S^{-1}AS = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

In fact, $B = [T(1)]_B [T(t)]_B [T(t^2)]_B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.