HONOR 2201, CALCULUS FOR NON-SCIENCE MAJORS, QUIZ 9, 12/02/05

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1-3. Let $p(t) = \frac{x}{18}$ be the density function for the shelf life of a brand of banana which lasts up to 6 weeks. Time t is measured in weeks and $0 \le t \le 6$.

1.(5 pts) What is the cumulative distribution function value $\mathbf{P}(2)$ that the shelf life of a brand of banana lasts up to 2 weeks?

$$P(2) = \int_{-\infty}^{2} p(t)\,dt = \int_0^2 \frac{x}{18}\,dx = \left[\frac{x^2}{36}\right]_0^2$$

$$= \frac{2^2}{36} - \frac{0^2}{36}$$

$$= \frac{4}{36} = \frac{1}{9} \quad (\approx 11.1\%)$$

2.(5 pts) Find the mean time of the shelf life of a brand of a banana.

$$\text{mean time} = \int_{-\infty}^{\infty} x\,p(x)\,dx = \int_0^6 x \cdot \frac{x}{18}\,dx = \int_0^6 \frac{x^2}{18}\,dx$$

$$= \left[\frac{x^3}{54}\right]_0^6 = \frac{6^3}{54} - \frac{0^3}{54}$$

$$= \boxed{4 \text{ weeks}}$$

So a banana can last average for 4 weeks.

3.(5 pts) Find the median time of the shelf life of a brand of a banana. (You might use $\sqrt{2} \approx 1.4$.)

Find T such that $\int_{-\infty}^{T} p(x)\,dx = 0.5$.

$$\int_0^T \frac{x}{18}\,dx = \frac{1}{2} \Leftrightarrow \left[\frac{x^2}{36}\right]_0^T = \frac{1}{2} \Leftrightarrow \frac{T^2}{36} - \frac{0^2}{36} = \frac{1}{2}$$

$$\Leftrightarrow \frac{T^2}{36} = \frac{1}{2} \Leftrightarrow T^2 = 18 \Leftrightarrow T = \sqrt{18} = \sqrt{9 \cdot 2}$$

$$\Rightarrow T = \sqrt{9} \cdot \sqrt{2} \approx 3 \cdot (1.4) = \boxed{4.2 \text{ weeks}}$$

So half bananas can last up to 4.2 weeks.