HW 8. Solution

(1) $n=3,\quad \Delta x = \frac{4-2}{3} = \frac{2}{3},\quad x_0 = 2,\quad x_1 = 2+\frac{2}{3} = \frac{8}{3},\quad x_2 = \frac{10}{3},\quad x_3 = \frac{12}{3} = 4.$

$$LS_3 = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x = \Delta x\left(f(x_0)+f(x_1)+f(x_2)\right)$$
$$= \frac{2}{3}\left(\left(2^2+3\cdot 2+2\right)+\left(\left(\frac{8}{3}\right)^2+3\cdot\frac{8}{3}+2\right)+\left(\left(\frac{10}{3}\right)^2+3\cdot\frac{10}{3}+2\right)\right)$$
$$= \frac{2}{3}\left(12+\frac{64}{9}+10+\frac{100}{9}+12\right) = \boxed{34.8148}$$

(2) $$RS_3 = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x = \frac{2}{3}\left(\left(\left(\frac{8}{3}\right)^2+3\cdot\frac{8}{3}+2\right)+\left(\left(\frac{10}{3}\right)^2+3\cdot\frac{10}{3}+2\right)+\left(4^2+3\cdot 4+2\right)\right)$$
$$= \frac{2}{3}\left(\frac{64}{9}+10+\frac{100}{9}+12+30\right) = \boxed{46.8148}$$

(3) $$\text{Approximation} = \frac{LS_3+RS_3}{2} = \boxed{40.8148}$$

(4) $n=5,\quad \Delta x = \frac{4-2}{5} = \frac{2}{5},\quad x_0 = 2,\quad x_1 = 2+\frac{2}{5} = \frac{12}{5},\quad x_2 = \frac{14}{5},\quad x_3 = \frac{16}{5},\quad x_4 = \frac{18}{5},\quad x_5 = \frac{20}{5} = 4.$

$$LS_5 = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$
$$= \frac{2}{5}\left(\left(2^2+3\cdot 2+2\right)+\left(\left(\frac{12}{5}\right)^2+3\cdot\frac{12}{5}+2\right)+\left(\left(\frac{14}{5}\right)^2+3\cdot\frac{14}{5}+2\right)+\left(\left(\frac{16}{5}\right)^2+3\cdot\frac{16}{5}+2\right)+\left(\left(\frac{18}{5}\right)^2+3\cdot\frac{18}{5}+2\right)\right)$$
$$= \frac{2}{5}(92.8) = \boxed{37.12}$$

(5) $$RS_5 = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x$$
$$= \boxed{44.32}$$

(6) $$\text{Approximation} = \frac{LS_5+RS_5}{2} = \boxed{40.72}$$

(7) $\Delta x = \frac{2}{n},\quad x_k = 2+\frac{2k}{n} \Rightarrow x_{k-1} = 2+\frac{2(k-1)}{n}$

$$LS_n = \sum_{k=1}^{n} f(x_{k-1})\Delta x = \frac{2}{n}\cdot\sum_{k=1}^{n}\left[\left(2+\frac{2(k-1)}{n}\right)^2+3\cdot\left(2+\frac{2(k-1)}{n}\right)+2\right]$$
$$= \frac{2}{n}\sum_{k=1}^{n}\left[4+\frac{8(k-1)}{n}+\frac{4(k-1)^2}{n^2}+6+\frac{6(k-1)}{n}+2\right]$$
$$= \frac{2}{n}\sum_{k=1}^{n}\left[4+\frac{8}{n}k-\frac{8}{n}+\frac{4}{n^2}k^2-\frac{8}{n^2}k+\frac{4}{n^2}+6+\frac{6}{n}k-\frac{6}{n}+2\right]$$
$$= \frac{2}{n}\sum_{k=1}^{n}\left[\left(\frac{4}{n^2}\right)k^2+\left(\frac{8}{n}-\frac{8}{n^2}+\frac{6}{n}\right)k+\left(4-\frac{8}{n}+\frac{4}{n^2}+6-\frac{6}{n}+2\right)\right]$$
$$= \frac{2}{n}\sum_{k=1}^{n}\left[\left(\frac{4}{n^2}\right)k^2+\left(\frac{14}{n}-\frac{8}{n^2}\right)k+\left(\frac{4}{n^2}-\frac{14}{n}+12\right)\right]$$
$$= \frac{2}{n}\left[\left(\frac{4}{\cancel{n^2}}\right)\cdot\frac{\cancel{n}(n+1)(2n+1)}{6}+\left(\frac{14}{n}-\frac{8}{n^2}\right)\cdot\frac{n(n+1)}{2}+\left(\frac{4}{n^2}-\frac{14}{n}+12\right)\cdot n\right]$$
$$= \frac{2}{n}\left[\frac{2}{3}\cdot\frac{2n^2+3n+1}{n}+7(n+1)-\frac{4}{n}(n+1)+\frac{4}{n}-14+12n\right]$$
$$= \frac{2}{n}\left[\frac{4n}{3}+2+\frac{2}{3n}+7n+7-4-\cancel{\frac{4}{n}}+\cancel{\frac{4}{n}}-14+12n\right]$$
$$= \frac{2}{n}\left[\frac{2}{3n}+\frac{61}{3}n-9\right] = \boxed{\frac{4}{3n^2}+\frac{122}{3}-\frac{18}{n}}$$

(8). $RS_n = \sum_{k=1}^{n} f(x_k)\Delta x = \frac{2}{n} \cdot \sum_{k=1}^{n} \left[\left(2+\frac{2k}{n}\right)^2 + 3\cdot\left(2+\frac{2k}{n}\right)+2\right]$

$= \frac{2}{n} \cdot \sum_{k=1}^{n} \left[4 + \frac{8k}{n} + \frac{4k^2}{n^2} + 6 + \frac{6k}{n} + 2\right]$

$= \frac{2}{n} \cdot \sum_{k=1}^{n} \left[\left(\frac{4}{n^2}\right)k^2 + \left(\frac{14}{n}\right)k + 12\right]$

$= \frac{2}{n} \left[\frac{4}{n^2} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{14}{n} \cdot \frac{n(n+1)}{2} + 12 \cdot n\right]$

$= \frac{2}{n} \left[\frac{2}{3} \cdot \frac{2n^2+3n+1}{n} + 7(n+1) + 12n\right]$

$= \frac{2}{n} \left[\frac{4}{3}n + 2 + \frac{2}{3n} + 7n + 7 + 12n\right] = \frac{2}{n}\left[9 + \frac{61}{3}n + \frac{2}{3n}\right]$

$= \boxed{\frac{4}{3n^2} + \frac{122}{3} + \frac{18}{n}}$

(9) Area $=$ $\lim_{n\to\infty} RS_n = \lim_{n\to\infty}\left[\cancelto{0}{\frac{4}{3n^2}} + \frac{122}{3} + \cancelto{0}{\frac{18}{n}}\right] = \frac{122}{3} = \boxed{40.667}$

(because the graph is above x-axis on [2, 4])

(10) The precise area $= 40.667$ and

Approximation in (3) has bigger difference with 40.667

than approximation in (6).

So 5-subdivision gives better approximation

than 3-subdivisions.