HW 7. Solution.

#1. $f(x) = x^3 + 6x - 3$, $\quad f(0) = -3 < 0$ & $f(1) = 4 > 0$. $\Rightarrow a_1 = 0,\ b_1 = 1$.

n	a_n	b_n	$f(a_n)$	$f(b_n)$	m_n	$f(m_n)$	h_n	
1	0	1	$-$	$+$	0.5	$+$	0.5	> 0.09
2	0	0.5	$-$	$+$	0.25	$-$	0.25	> 0.09
3	0.25	0.5	$-$	$+$	0.375	$-$	0.125	> 0.09
4	0.375	0.5	$-$	$+$	0.4375	$-$	0.0625	< 0.09.

A Solution is approximately $\boxed{0.4375}$

#2. $f'(x) = 3x^2 + 6$. Let $x_1 = 0$. & $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

n	x_n	$h_n = \lvert x_n - x_{n+1} \rvert$
1	0	
2	0.5	$0.5 > 0.09$
3	0.4815	$0.0185 < 0.09$

So approximately $\boxed{0.4815}$.

#3. (1) $$\sum_{k=1}^{20} (k^2 - k + 2) = \sum_{k=1}^{20} k^2 - \sum_{k=1}^{20} k + \sum_{k=1}^{20} 2$$

$$= \frac{20\cdot(20+1)(2\cdot 20+1)}{6} - \frac{20\cdot(20+1)}{2} + 2\cdot 20 = 2700$$

(2) $$\sum_{k=10}^{30} 3\left(\tfrac{1}{2}\right)^k = 3\cdot\left(\tfrac{1}{2}\right)^{10} + 3\cdot\left(\tfrac{1}{2}\right)^{11} + 3\cdot\left(\tfrac{1}{2}\right)^{12} + \cdots + 3\cdot\left(\tfrac{1}{2}\right)^{30}$$

$$= \boxed{\frac{3\cdot\left(\frac{1}{2}\right)^{10}\left(\left(\frac{1}{2}\right)^{21} - 1\right)}{\frac{1}{2} - 1}}$$

the same!

$$\left(= \sum_{k=1}^{30} 3\cdot\left(\tfrac{1}{2}\right)^k - \sum_{k=1}^{9} 3\cdot\left(\tfrac{1}{2}\right)^k = \boxed{\frac{3\cdot\frac{1}{2}\left(\left(\frac{1}{2}\right)^{30} - 1\right)}{\frac{1}{2} - 1} - \frac{3\cdot\frac{1}{2}\left(\left(\frac{1}{2}\right)^{9} - 1\right)}{\frac{1}{2} - 1}} \right)$$