NAME:____________________

MATH 1180
The Final

Do all five problems, each worth 40 points. Make sure I can find both the answer and how you got it. No calculators. Each extra credit problem is worth 2 points.

1. Suppose they finally invent those vacuuming robots (VRs). One brand of VR switches from the active state (vacuuming) to the inactive state (resting) at rate 0.4/hour, and from the inactive state to the active state at rate 1.2/hour. Let A represent the probability a VR is in the active state, and I the probability it is in the inactive state.

- **a.** Write a pair of coupled differential equations for A and I. Draw a little diagram with circles and arrows to help yourself if needed.
- **b.** Use the fact that $A + I = 1$ to write a single differential equation for A.
- **c.** Draw the phase-line diagram, find the equilibrium, and say whether it is stable.
- **d.** If there were 100 of these VRs operating independently in a large mall, what distribution (with what parameter values) describes the number that are active after they have been running for a long time?

Extra credit: Name one speaker at the Utah Symposium on Science and Literature

2. Suppose that VRs wander around the room randomly, and clean each point on the floor independently and at the same rate. To test VR efficiency, 5 points are watched with remote video hook-ups for one day and the number of visits by the VR counted. Of those 5 points, two received 1 visit, one received 2 visits, and two received 3 visits.

a. What probability distribution describes the number of visits to each point?

b. Using the data, write the likelihood function for the parameter of this distribution.

c. Find the maximum likelihood estimator for this parameter.

d. Write the equation you would solve to find the confidence limits with the method of support.

e. What is your best guess of the probability that a point on the floor is entirely missed?

Extra credit: Compute 0^0.

3. VRs seem to do better in rooms with fewer obstacles, such as chairs, children, and cats. Five rooms are measured for number of obstacles N and fraction of area A missed.

Number of obstacles N	Area missed A
1	0.1
1	0.2
3	0.3
3	0.4
5	0.5

a. Find SSE for the model $A = 0.1N$.

b. Find SST for the null model that obstacles have no effect on area missed.

c. Find r^2 for the model $A = 0.1N$.

d. Graph the data and the model, and sketch your guess of the best fit line.

Extra credit: What is the Maple command to numerically solve an equation?

4. Careful consumers buy two brands of VR and set them up in the same room to try to increase vacuuming efficiency (and see if they will fight). Each brand vacuums 80% of the room each day and misses 20%. However, 10% of the room is missed by both.

- **a.** Find the joint distribution for the probability a spot is vacuumed by both, by just brand 1, just brand 2, or by neither.
- **b.** Find the two conditional distributions for brand 1.
- **c.** If the two brands were independent, how much area would be missed by both?
- **d.** Suppose that the room is made up of hard to get spots (40% of the area) and easy spots (the remaining 60%). The two brands behave independently and identically in these two regions, and each always vacuums the entire easy part. What fraction of the hard part does each vacuum?

Extra credit: What country won the most medals the 2002 Winter Olympics?

5. The advertising DVD distributed with a popular brand of VR claims that waiting times between successive visits to a point are exponentially distributed with a mean of 2.0 days (in a 250 ft^2 room). While watching the DVD with your partner, you remark loudly "I bet that waiting times will be longer than that". You then measure the first 36 waiting times, finding a sample mean of 2.4.

a. If waiting times are indeed exponentially distributed, what is a good guess of the sample standard deviation?

b. Use this value to find the standard error of the mean.

c. Find the p-value of the appropriate statistical test. Would you write a steaming letter and return the VR based on your results?

d. About how many waiting times would you have to observe to get a result significant at the 0.05 level?

Extra credit: Create an acronym and mnemonic for your favorite method from this class.

Areas under the standard normal curve

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-4.0	0.00003	-2.0	0.02275	0.0	0.50000	2.0	0.97725
-3.9	0.00005	-1.9	0.02872	0.1	0.53983	2.1	0.98214
-3.8	0.00007	-1.8	0.03593	0.2	0.57926	2.2	0.98610
-3.7	0.00011	-1.7	0.04457	0.3	0.61791	2.3	0.98928
-3.6	0.00016	-1.6	0.05480	0.4	0.65542	2.4	0.99180
-3.5	0.00023	-1.5	0.06681	0.5	0.69146	2.5	0.99379
-3.4	0.00034	-1.4	0.08076	0.6	0.72575	2.6	0.99534
-3.3	0.00048	-1.3	0.09680	0.7	0.75804	2.7	0.99653
-3.2	0.00069	-1.2	0.11507	0.8	0.78814	2.8	0.99744
-3.1	0.00097	-1.1	0.13567	0.9	0.81594	2.9	0.99813
-3.0	0.00135	-1.0	0.15865	1.0	0.84134	3.0	0.99865
-2.9	0.00187	-0.9	0.18406	1.1	0.86433	3.1	0.99903
-2.8	0.00256	-0.8	0.21185	1.2	0.88493	3.2	0.99931
-2.7	0.00347	-0.7	0.24196	1.3	0.90320	3.3	0.99952
-2.6	0.00466	-0.6	0.27425	1.4	0.91924	3.4	0.99966
-2.5	0.00621	-0.5	0.30854	1.5	0.93319	3.5	0.99977
-2.4	0.00820	-0.4	0.34458	1.6	0.94520	3.6	0.99984
-2.3	0.01072	-0.3	0.38209	1.7	0.95543	3.7	0.99989
-2.2	0.01390	-0.2	0.42074	1.8	0.96407	3.8	0.99993
-2.1	0.01786	-0.1	0.46017	1.9	0.97128	3.9	0.99995