MATH 1180
MATHEMATICS FOR LIFE SCIENTISTS
Computer Assignment VIII
Due April 6, 2004

Warm up Maple for today's problems with the commands

```
> with(stats);
> iread(histplot);
> iread(binomial);
> iread(Poisson);
> iread(iter);
```

The "Poisson" package contains three commands. The first, typed as

```
> Poi(k,Lambda);
```

gives the value of the Poisson probability distribution denoted $p(k; \Lambda)$, the probability of exactly k events with parameter Λ. The second, typed as

```
> Poicum(k1,k2,Lambda);
```

gives the cumulative probability of between `k1` and `k2` events (inclusive). The last, typed

```
> Poiplot(Lambda,N);
```

plots out the Poisson probability distribution with parameter Λ up to a maximum of N (because the theoretical maximum is infinite).

PROBLEMS

- **1.** Suppose events occur at rate $\lambda = 0.25$. Maple has a command to generate values from an exponentially distributed random variable T with the command

  ```
  > u:=x -> random[gamma[1,1/lambda]](1);
  ```

  The **gamma** distribution is a generalized version of the exponential distribution, and the first 1 tells Maple to use the exponential. Notice that the second parameter is $1/\lambda$. The function u can be thought of as an updating function. For later use, pick a value and set _seed equal to this value. Use `iterplot` for 20 steps. Explain your graph. Where is the theoretical mean and standard deviation?

- **2.** The exponentially distributed random variable T describes the waiting time between events. If the nth event occurs at time t, the $n + 1$st event occurs at time $t + T$. The updating function describing this process is

  ```
  > g := t -> t+u();
  ```

  Reset _seed to your chosen value, and run `iterplot` using the function g for 20 steps. How many events occurred by time 20? Using the picture from problem 1, mark the two longest waits and their durations. Using your computation of the average waiting time, sketch the line that should lie close to your graph.

- **3.** The number of events at time 20 should be described by a Poisson distribution with parameter $\Lambda = 20\lambda$. Use the `Poiplot` command to plot this distribution up to a reasonable upper bound N. Using the number of events that occurred by time 20 in problem 2 as k, compute the probability of exactly k, fewer than k, and more than k events (for the last pick a reasonable upper bound `k2`). Indicate the areas associated with each of these events on your graph. Do you consider your result from problem 2 peculiar?

- **4.** Find a binomial distribution that approximates the Poisson in the previous problem. Plot it with the `binplot` command. How close are the two distributions? Compare the probabilities at a couple of points. Which distribution has lower variance and why?