MATH 1180
MATHEMATICS FOR LIFE SCIENTISTS
Computer Assignment V
Due February 17, 2004

Maple has several features for simulating stochastic dynamical systems. In particular, it has various so-called "random number generators". For example, to get Maple to pick a random integer between 1 and 6, use the command

```
> die := rand(1..6);
```

Maple responds with a bunch of incomprehensible garbage that defines `die` to be a function with no arguments (except Lady Luck). To get actual numbers type

```
> die();
```

Roll your computer die a few times. What criteria might you use to check whether the rolls are "fair"?

To get Maple to pick a "random number" between 0 and 1, use the commands

```
> with(stats);
> number := x -> stats[random,uniform[0,1]](1);
```

The first command tells Maple to use its library of statistical commands and the second defines a function rather like `die`. To see your numbers, type

```
> number();
```

a few times.

PROBLEMS

- **1.** Consider a population of beavers grows according to the rule

  $$b_{t+1} = rb_t$$

  where r is a "random number" chosen from the range 0.5 to 1.5. To define the updating function f, redefine `number` to take on values in the range from 0.5 to 1.5 (by replacing the `[0,1]` with `[0.5,1.5]`) and set

  ```
  > f := b -> number()*b;
  ```

  This function gives the new population as a function of the old population and the value of the growth parameter. To study the dynamics, revive the procedure "iter" with the command

  ```
  > iread(iter);
  ```

  The population after one generation starting from 100 is $f(100)$. Type this a few times. To see a plot of the population for 50 generations starting from 100, type

  ```
  > iterplot(f,50,100.);
  ```

  To see plots of two populations, type

```
> iterplot2(f,f,50,100.,100.);
```

Try this a few times until you get a plot that looks interesting. Print this graph and label the final size of each population. Why are the two solutions different? If someone showed you these data without telling you they were generated on a computer, how would you describe and interpret the results?

- **2.** Consider a population growing due to immigration

$$N_{t+1} = \begin{cases} N_t + 1 & \text{with probability } 0.5 \\ N_t & \text{with probability } 0.5. \end{cases}$$

A special function `bern` (named after one of the famous family of French mathematicians, the Bernoulli's) has been defined to simulate this situation, and can be read in with

```
> iread(draw);
```

`bern` is a function of p that gives a result 1 with probability p and a result 0 with probability $1 - p$ when you type `bern(p);`. Try "bern(0.5)", "bern(0.1)" and "bern(0.9)" a few times to see how they work.

Define an updating function g

```
> g := N -> N + bern(0.5);
```

and use `iterplot2` to generate two 50 generation solutions starting from populations of 0. Print the graph and label the final size of each population. If someone showed you this as data, how would you describe and interpret the results? Are your solutions more similar than those in **1**. Do the two populations ever cross? Does this have anything to do with basketball?

- **3.** Consider a population of lizards on an island described by $M = 1$ if the island is occupied and $M = 0$ if the island is unoccupied. Suppose this population follows the rule

$$M_{t+1} = 1 \begin{cases} \text{with probability } 0.3 \text{ if } M_t = 0 \\ \text{with probability } 0.9 \text{ if } M_t = 1 \end{cases}$$

$$M_{t+1} = 0 \begin{cases} \text{with probability } 0.7 \text{ if } M_t = 0 \\ \text{with probability } 0.1 \text{ if } M_t = 1. \end{cases}$$

There is a tricky way to program the updating function in Maple as

```
> h := M -> M*bern(0.9)+(1-M)*bern(0.3);
```

If $M = 1$, $h = \text{bern}(0.9)$ and M remains at 1 with probability 0.9, while if $M = 0$, $h = \text{bern}(0.3)$ and M switches to 1 with probability 0.3. Use `iterplot` to generate a solution of 50 generations of this population starting from the occupied state. When does the population first go extinct? For how long? What is the final state of the population? Is the island occupied more often than unoccupied? Does this make sense?

- **4.** Consider again the population of lizards in **3**. The **probability** p that the island is occupied follows the updating function

```
> H := p -> 0.9*p+0.3*(1-p);
```

This matches the equation for $h(M)$ on "average". Use `iterplot2` to plot a solution of the probability equation and a simulation of an island starting from $M = p = 1$ (use enough steps to see what is going on). Solve for the equilibrium probability. Why doesn't the simulation seem to approach an equilibrium? What do the two graphs have to do with each other?