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Mathematics 1180
MATHEMATICS FOR LIFE SCIENTISTS
Computer Assignment II
Due January 27, 2004

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Exercise 1 Use the previous assignment to recall the method for getting Maple to solve autonomous differential equations. For example, consider the equation

$$\frac{dp}{dt} = 3p(1 - p).$$

The following commands will find the solution $P(t)$ starting from the initial condition $p(0) = 0.01$.

```
> de := {diff(p(t),t)=3*p(t)*(1-p(t)),p(0)=0.01};
> P := unapply(rhs(dsolve(de,p(t))),t);
```

a. Give values of μ and λ in the equation for competing bacteria that would produce this equation. (don't forget units)

b. How long will it take the fraction to reach 0.99? (what are the units)

c. Plot the solution $P(t)$.

d. Check your solution (show that the derivative is really proportional to $P(t)(1 - P(t))$.

Exercise 2 An amusing class of differential equations looks like

$$\frac{db}{dt} = b^m.$$

for some power m. Suppose that the initial condition is $b(0) = 1.0$.

a. Have Maple solve the equation with $m = 0.5$, $m = 1.0$ and $m = 2.0$. It might do something weird when $m = 1.0$ – but use your brain or the `limit` command to figure out what the solution ought to be. Graph the solutions as **semilog graphs** (by graphing `log(B(t))`) and describe them in words. What happens to the case with $m = 2.0$ at $t = 1.0$?

b. What is the per capita reproduction as a function of population size in each of these cases? What might it mean biologically?

c. Remember Euler's method? Write down equation for Euler's method and input it as a discrete-time dynamical system (use $\Delta t = 1$). What do the solutions do? How does Euler's method deal with the fact that the $m = 2$ case blows up?

Exercise 3 Consider the chemical diffusion problem assuming that the external concentration oscillates with period T according to

$$?(t) = 20.0 + \sin(\frac{2\pi t}{T}).$$

When you input this, make sure to type π as `Pi` with a capital P. The differential equation is

$$\frac{dC}{dt} = \alpha(?(t) - C).$$

Set $T = 1$, and assume the initial condition is $C(0) = 20\mu moles/cm^3$.

a. Use Maple to find the solution (it should be able to do it without setting particular parameter values).

b. Plot $C(t)$ and $?(t)$ using values of α ranging from 0.1 to 100.0. Make sure to plot enough of the solution to see what is happening. When does the internal concentration of the chemical most closely track the external concentration? Why?