Problem 5. A graphic called a **phase diagram** displays the behavior of all solutions of $u' = F(u)$. A **phase line diagram** is an abbreviation for a direction field on the vertical axis (u-axis). It consists of equilibrium points and signs of $F(u)$ between equilibria. A phase diagram can be created solely from a phase line diagram, using just three drawing rules:

1. Solutions don't cross.

2. Equilibrium solutions are horizontal lines $u = c$. All other solutions are increasing or decreasing.

3. A solution curve can be moved rigidly left or right to create another solution curve.

Use these tools on the equation $u' = (u-1)(u-2)^2(u+2)$ to make a phase line diagram, and then make a phase diagram with at least 8 threaded solutions. Label the equilibria as stable, unstable, funnel, spout, node.

References. Edwards-Penney section 2.2.
Course document on **Stability**,:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250phaseline.pdf`

Problem 6. An autonomous differential equation $\frac{dy}{dx} = F(x)$ with initial condition $y(0) = y_0$ has a formal solution

$$y(x) = y_0 + \int_0^x F(u)du.$$

The integral may not be solvable by calculus methods. In this case, the integral is evaluated numerically to compute $y(x)$ or to plot a graphic. There are three basic numerical methods that apply, the rectangular rule (RECT), the trapezoidal rule (TRAP)and Simpson's rule (SIMP).

Apply the three methods for $F(x) = \cos(x^2)$ and $y_0 = 0$ using step size $h = 0.2$ from $x = 0$ to $x = 1$. Then fill in the blanks in the following table. Use technology if it saves time. Lastly, compare the four data sets in a plot, using technology.

$x-$values	0.0	0.2	0.4	0.6	0.8	1.0
$y-$ to 10 digits	0.0	0.1999680024	0.3989772129	0.5922705167	0.7678475376	0.9045242379
$y-$RECT values	0.0	0.2	0.3998400213	0.5972854780		0.9448839943
$y-$TRAP values	0.0	0.1999200107	0.3985627497		0.7646744186	0.8989142250
$y-$SIMP values	0.0	0.1999666703	0.3989746144	0.5922670741	0.7678445414	

References. Edwards-Penney Sections 2.4, 2.5, 2.6, because methods Euler, Modified Euler and RK4 reduce to RECT, TRAP, SIMP methods when $f(x, y)$ is independent of y, i.e., an equation $y' = F(x)$.
Course document on numerical solution of $y' = F(x)$, **RECT, TRAP, SIMP methods**:
`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/solve-quadrature-numerically.pdf`
Wolfram Alpha at `http://www.wolframalpha.com/` can do the RECT rule and graphics with input string

```
integrate cos(x^2) using left endpoint method with interval width 0.2 from
x=0 to x=1
```