Instructions. Please prepare your own report on 8x11 paper, handwritten. Work alone or in groups. Help is available by telephone, office visit or email. All problems in Part 1 of the semester project reference Chapters 1, 2 in Edwards-Penney. Use the sample problems with solutions to fully understand the required details:

`http://www.math.utah.edu/~gustafso/s2019/2280/quiz/sampleQuizzes/project-part1.pdf`

Visit the Math Center in building LCB for assistance on problem statements, references and technical details.

Problem 1. An answer check for the differential equation and initial condition

$$\frac{dy}{dx} = k(73 - y(x)), \quad y(0) = 28 \tag{1}$$

requires substitution of the candidate solution $y(x) = 73 - 45\,e^{-kx}$ into the left side (LHS) and right side (RHS), then compare the expressions for equality for all symbols. The process of testing LHS = RHS applies to both the differential equation and the initial condition, making the answer check have **two** presentation panels. Complete the following:

1. Show the two panels in an answer check for initial value problem (1).

2. Relate (1) to a Newton cooling model for warming a 28 F frozen ice cream bar to room temperature 73 F.

3. Let x be the time in minutes. Find the Newton cooling constant k, given the additional information that the ice cream bar reaches 33 F in 5 minutes.

References. Edwards-Penney sections 1.1, 1.4, 1.5. Newton cooling in Serway and Vuille, *College Physics 9/E*, Brooks-Cole (2011), ISBN-10: 0840062060. Newton cooling differential equation $\frac{du}{dt} = -h(u(t) - u_1)$, Math 2280 slide **Three Examples**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250ThreeExamples.pdf`

Math 2280 slide on **Answer checks**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/FTC-Method-of-Quadrature.pdf`

Problem 2. A 2-ft high conical water urn drains from an orifice 6 inches above the base. The tank drains according to the Torricelli model

$$|y(x)|^2 \frac{dy}{dx} = -0.021\sqrt{|y(x)|}, \quad y(0) = y_0. \tag{2}$$

Symbol $y(x) \geq 0$ is the tank water height in feet above the orifice at time x seconds, while $y_0 \geq 0$ is the water height at time $x = 0$.

Establish these facts about the physical problem.

1. If $y_0 > 0$, then the solution $y(x)$ is uniquely determined and computable by numerical software. Justify using Picard's existence-uniqueness theorem.

2. Solve equation (2) using separation of variables when y_0 is 19 inches, then numerically find the drain time. Check your answer with technology.

References. Edwards-Penney, Picard's theorem 1 section 1.3 and Torricelli's Law section 1.4. Tank draining `Mathematica` demo at **Wolfram Research**:
http://demonstrations.wolfram.com/TimeToDrainATankUsingTorricellisLaw/
Carl Schaschke, *Fluid Mechanics: Worked Examples for Engineers*, The Institution of Chemical Engineers (2005), ISBN-10: 0852954980, Chapter 6.
Math 2280 slide on **Picard and Peano Theorems**:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/Picard+DirectionFields.pdf`

Manuscript on applications of first order equations Example 35:

`http://www.math.utah.edu/~gustafso/s2019/2280/lectureslides/2250SciEngApplications.pdf`,