Course Content

- First Order Differential Equations
- First Order Applications and Numerical Methods
- Linear Nth Order Differential Equations
- Mechanical and Electrical Applications
- Systems of Differential Equations
- Dynamical Systems
- Laplace Transform Theory
- Fourier Series
- Heat and Wave Partial Differential Equations

Details appear in the slides below, one for each title.

First Order Differential Equations

Quadrature, linear and separable equations are studied.

Three special methods are introduced for linear equations.

Topics: quadrature equation, separable equation, linear equation, direction field, equilibria, phase diagram, Picard–Lindelöf and Peano theorems, No-Crossing theorem, Exponential Growth-Decay model, Newton cooling, brine tank, Torricelli's law.

Key Methods. Method of quadrature, separable equation algorithm, linear integrating factor method, linear homogeneous shortcut, linear constant-coefficient shortcut, superposition shortcut.

First Order Applications and Numerical Methods

Population models, equilibrium, phase diagrams and stability. Newton linear and nonlinear models for kinematics. Numerical methods for $\boldsymbol{y' = F(x)}$ and $\boldsymbol{y' = f(x, y)}$.

Dynamics Topics: Malthus and Verhulst-logistic modeling, US population predictions, carrying capacity, fish populations, explosion and extinction, stable and unstable equilibria, speed governor, phase diagram, funnel, spout, node, harvesting, gravitation law, crossbow problem, Jules Verne escape velocity of the earth.

Numerical Topics: Rectangular rule, Trapezoidal Rule, Simpson's Rule, Euler's Method, Heun's Method, Runge-Kutta Method.

Key Methods: Separation of variables, logistic solution formula, phase line diagram, sign analysis, increasing and decreasing functions, bifurcation analysis, translation of a graphic, computer codes for numerical methods.

Linear Nth Order Differential Equations

Second order linear differential equation $ay'' + by' + cy = 0$, Picard theorem, spring-mass system, LRC-circuit, Euler's substitution, general solution, independence, Wronskian test, Abel formula.

Nth order linear differential equation $p_0 y^{(N)} + \cdots + p_n y = 0$, superposition, general solution, Picard existence-uniqueness, Wronskian test, Abel's independence theorem, characteristic equation, companion system, algorithm to solve for the general solution.

Mechanical vibrations, undamped pendulum, damped oscillations, undetermined coefficients, variation of parameters, forced oscillations, spring-mass system with forcing term, beats, resonance pure and practical, transients, steady-state, phase-amplitude form, electric circuits.

Main Theorems: Picard existence-uniqueness, Abel's formula, Wronskian test, Euler's substitution theorems, Algorithm for the general solution using Euler solution atoms, undetermined coefficient algorithm, variation of parameters formula, Steady-state formula, Resonance formula.

Mechanical and Electrical Applications

To be continued . . .

Systems of Differential Equations

To be continued . . .

Dynamical Systems

To be continued ...

Laplace Transform Theory

To be continued . . .

Fourier Series

To be continued ...

Heat and Wave Partial Differential Equations

To be continued . . .