

```
> # Example 3, Markov Chains
> P:=(0.1)*<5,2,3|3,8,3|2,0,4>^+;
```

$$P := \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.8 & 0.3 \\ 0.2 & 0. & 0.4 \end{bmatrix} \quad (1)$$

```
> x[0]:=<1,0,0>; # initial state
```

$$x_0 := \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

```
> x[1]:=P.x[0]; # state after one time step
```

$$x_1 := \begin{bmatrix} 0.5000000000000000 \\ 0.3000000000000000 \\ 0.2000000000000000 \end{bmatrix} \quad (3)$$

```
> x[2]:=P.x[1]; # state after two time steps
```

$$x_2 := \begin{bmatrix} 0.3700000000000000 \\ 0.4500000000000000 \\ 0.1800000000000000 \end{bmatrix} \quad (4)$$

```
> x[3]:=P.x[2]; # state after three time steps
```

$$x_3 := \begin{bmatrix} 0.3290000000000000 \\ 0.5250000000000000 \\ 0.1460000000000000 \end{bmatrix} \quad (5)$$

```
> # Briefly, x[n] = P^n . x[0];
> P^3 . x[0];
```

$$\begin{bmatrix} 0.3290000000000000 \\ 0.5250000000000000 \\ 0.1460000000000000 \end{bmatrix} \quad (6)$$

```
> # Long term guess:
P^300 . x[0]
```

$$\begin{bmatrix} 0.3000000000000006 \\ 0.6000000000000013 \\ 0.1000000000000002 \end{bmatrix} \quad (7)$$

```
> # Steady-state or equilibrium vector
q:=<.3,.6,.1>;
```

$$q := \begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix} \quad (8)$$

```
> P.q-q; # Conclude P.q=q
```

$$\begin{bmatrix} 5.55111512312578 \cdot 10^{-17} \\ 0. \\ 0. \end{bmatrix}$$

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> with(LinearAlgebra):
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```
> # Try to find q by solving P.q=q
```

```
> A:=P-1;LinearSolve(A,<0,0,0>);Rank(A);ReducedRowEchelonForm(A);
```

$$A := \begin{bmatrix} -0.5000000000000000 & 0.2000000000000000 & 0.3000000000000000 \\ 0.3000000000000000 & -0.2000000000000000 & 0.3000000000000000 \\ 0.2000000000000000 & 0. & -0.6000000000000000 \end{bmatrix}$$

$$\begin{bmatrix} -0. \\ 0. \\ 0. \end{bmatrix}$$

2

$$\begin{bmatrix} 1. & -0. & -0. \\ 0. & 1. & 0. \\ 0. & 0. & 1. \end{bmatrix}$$

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```
> # Did not work, failed to find q:=<.3,.6,.1>;
```

```
> # Get rid of decimal approximations
```

```
PP:=(1/10)*<5,2,3|3,8,3|2,0,4>^+;
```

$$PP := \begin{bmatrix} \frac{1}{2} & \frac{1}{5} & \frac{3}{10} \\ \frac{3}{10} & \frac{4}{5} & \frac{3}{10} \\ \frac{1}{5} & 0 & \frac{2}{5} \end{bmatrix}$$

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```
> AA:=PP-1;Rank(AA);ReducedRowEchelonForm(AA);LinearSolve(AA,<0,0,0>,free='t');
```

$$AA := \begin{bmatrix} -\frac{1}{2} & \frac{1}{5} & \frac{3}{10} \\ \frac{3}{10} & -\frac{1}{5} & \frac{3}{10} \\ \frac{1}{5} & 0 & -\frac{3}{5} \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 t_3 \\ 6 t_3 \\ t_3 \end{bmatrix}$$

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```
> # To make the answer a stochastic vector, choose t=1/10  
> qq:=subs(t[3]=1/10,%);evalf(qq);
```

$$qq := \begin{bmatrix} \frac{3}{10} \\ \frac{3}{5} \\ \frac{1}{10} \end{bmatrix}$$

$$\begin{bmatrix} 0.3000000000 \\ 0.6000000000 \\ 0.1000000000 \end{bmatrix}$$

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