

Strang: Chapter 2

Section 2.1. Exercises 9, 10, 21, 29, 34

Section 2.2. Exercises 5, 9, 12, 13, 32

Section 2.3. Exercises 3, 13, 19, 25, 27, 28

Section 2.4. Exercises 2, 5, 22, 24

Section 2.5. Exercises 1, 7, 8, 14, 15, 22, 24, 40

Section 2.6. Exercises 6, 12, 15, 23

Section 2.7. Exercises 2, 7, 20, 40

Some Answers

2.1. Exercises 29, 34 have answers in the textbook. Reproduced here.

2.1-9. (a) $Ax = (18, 5, 0)$ and (b) $Ax = (3, 4, 5, 5)$.

2.1-10. Multiplying as linear combinations of the columns gives the same Ax . By rows or by columns: 9 separate multiplications for 3 by 3.

2.1-21. $R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. The entries are sines and cosines of 45 degrees.

2.1-29. $u_2 = \begin{pmatrix} .7 \\ .3 \end{pmatrix}$ and $u_3 = \begin{pmatrix} .65 \\ .35 \end{pmatrix}$. The components add to 1. They are always positive. u_7, v_7, w_7 are all close to $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$. Their components still add to 1.

2.1-34. $A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$ and $A\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ has solution $\vec{x} = \begin{pmatrix} 4 \\ 7 \\ 8 \\ 6 \end{pmatrix}$.

2.2. Exercises 5, 9, 12, 13, 32 do not have textbook answers. Below for answers.

2.2-5. $6x + 4y$ is 2 times $3x + 2y$. There is no solution unless the right side is $2 \times 10 = 20$. Then all the points on the line $3x + 2y = 10$ are solutions, including $(0, 5)$ and $(4, -1)$. (The two lines in the row picture are the same line, containing all solutions).

2.2-9. On the left side, $6x - 4y$ is 2 times $3x - 2y$. Therefore we need $b_2 = 2b_1$ on the right side. Then there will be infinitely many solutions (two parallel lines become one single line).

2.2-12. Elimination first gives an upper triangular system

$$\begin{aligned} 2x + 3y + z &= 8 \\ y + 3z &= 4 \\ 8z &= 8 \end{aligned}$$

Back-substitution sorts out the answer. Consider two possibilities: (a) y is a free variable, (b) z is a free variable.

2.2-13. Whatever the elimination details, $x = 3, y = 1, z = 0$.

2.2-32. The question deals with 100 equations $Ax = 0$ when A is singular. (a) Some linear combination of the 100 rows is the row of 100 zeros.

(b) Some linear combination of the 100 columns is the column of zeros.

(c) A very singular matrix has all ones: $A = eye(100)$. A better example has 99 random rows (or the numbers $1^i, \dots, 100^i$ in those rows). The 100th row could be the sum of the first 99 rows (or any other combination of those rows with no zeros).

(d) The row picture has 100 planes meeting along a common line through 0. The column picture has 100 vectors all in the same 99-dimensional hyperplane.

2.3. Exercises 3, 25, 27, 28 have a textbook answer.

2.3-13. (a) E times the third column of B is the third column of EB . A column that starts at zero will stay at zero. (b) E could add row 2 to row 3 to change a zero row to a nonzero row.

2.3-19. Any swap elementary matrix S satisfies $S^2 = I$.

2.4. All exercises 2, 5, 22, 24 have a textbook answer.

2.5. Exercises 1, 7, 8, 14, 15, 22, 24 have a textbook answer.

2.5-40. The inverse of $A = \begin{pmatrix} 1 & -a & 0 & 0 \\ 0 & 1 & -b & 0 \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{pmatrix}$ is $A^{-1} = \begin{pmatrix} 1 & a & ab & abc \\ 0 & 1 & b & bc \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$. See also the cofactor formula

for the inverse in 5.3.

2.6. All exercises 6, 12, 15, 23 have a textbook answer.

2.7. Exercises 2, 20 are in the textbook.

2.7-7. (a) False, (b) False, (c) True, (d) True. Supply reasons and proofs.

2.7-40. Start from $Q^T Q = I$, as in $\begin{pmatrix} q_1^T \\ q_2^T \end{pmatrix} \begin{pmatrix} q_1 & q_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The diagonal entries give $q_1^T q_1 = 1$ and $q_2^T q_2 = 1$, unit vectors.

(b) The off-diagonal entry is $q_1^T q_2 = 0$ (and in general $q_i^T q_j = 0$).

(c) The leading example for Q is the rotation matrix $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$.