Name ________________________ Class Time _____

Math 2270 Extra Credit Problems
Chapter 4
December 2011

These problems were created for Bretscher's textbook, but apply for Strang's book, except for the division by chapter. To find the background for a problem, consult Bretscher's textbook, which can be checked out from the math library or the LCB Math Center.

Due date: See the internet due dates. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem XC4.1-20. (Matrix space basis)

Find a basis and the dimension for the space of all 2×2 matrices $\begin{pmatrix} a & b \\ a+b & a-b \end{pmatrix}$.

Problem XC4.1-26. (Polynomial space basis)

Find a basis and the dimension for the space of all polynomials $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ satisfying the conditions $p(-1) = p(1)$, $\int_0^1 p(x)dx = p(0)$.

Problem XC4.1-38. (Commutator dimension)

Find the dimension of the space of all 3×3 matrices A that commute with a 3×3 diagonal matrix B.

Problem XC4.1-54. (Subspace dimension)

Let V be a vector space of dimension n and let S be a subspace of V. Prove that S has a basis of k elements and $k \le n$.

Problem XC4.2-13. (Linear transformations and isomorphisms)

Let $T(A) = PA - AP$ where $P = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$. Prove that T is a linear transformation on the set V of all 2×2 matrices A. Determine the kernel and image of T and report whether T is an isomorphism.

Problem XC4.2-20. (Kernel of a linear transformation)

Let $T(y) = 3y'' + 5y'$ be defined on the vector space V of all twice continuously differentiable functions $y(x)$ defined on $-\infty < x < \infty$ with range in some vector space W. Compute the kernel of T and therefore show that T is not an isomorphism. How should W be defined? Is there a problem with $W = V$?

Hint: Write $v = y'$ and break the equation $3y'' + 5y' = 0$ into $3v' + 5v = 0$ and $y' = v$. These are elementary first order differential equations. The general solution of $3v' + 5v = 0$ is $v = v_0 e^{-5x/3}$.

Problem XC4.2-34. (Sequence spaces)

Let $T(f)$ denote the sequence $f(0)$, $f'(0)$, $\ldots$ of derivatives of f evaluated at 0. Let V be the space of all polynomials. Then the vector space W is the set of all sequences with only finitely many nonzero terms. Prove that W is a subspace of the vector space E of all real sequences equipped with termwise addition and scalar multiplication. Prove that T is linear and decide if it is an isomorphism.

Problem XC4.3-2. (Independence in matrix spaces)

Prove or disprove: a list of 2×2 matrices of length 5 or more is dependent.

Problem XC4.3-30. (Tangents)

Let $T(p) = p(0) + xp'(0)$ be defined on the polynomials $p(x) = a_0 + a_x + a_2 x^2$ of degree 2 or less, a vector space V. Prove that T is linear and compute the matrix of T relative to the basis 1, $x - 1$, $(x-1)^2$.

Problem XC4.3-60. (Matrix of change of basis)

Let a plane P be given in $\mathcal{R}^3$ having two different bases $\{\mathbf{v}_1, \mathbf{v}_2\}$ and $\{\mathbf{w}_1, \mathbf{w}_2\}$. Give general formulas for the change of basis matrices V and W which change one basis into the other (in both possible ways). Then give a formula which relates V to $\mathbf{aug}(\mathbf{v}_1, \mathbf{v}_2)$ and $\mathbf{aug}(\mathbf{w}_1, \mathbf{w}_2)$.

Problem XC4.3-64. (Matrix of a linear transformation)

Let $T(A) = PA - AP$ where $P = \begin{pmatrix} 2 & 3 \\ 0 & 7 \end{pmatrix}$. Prove that T is a linear transformation on the set V of all 2×2 upper triangular matrices A. (a) Find the standard basis $\mathcal{B}$ of V. (b) Find the matrix of T relative to basis $\mathcal{B}$. (c) Determine the rank of T.

End of extra credit problems chapter 4.