Linear Algebra 2270-2

Due in Week 5

The fifth week finishes the work from chapter 1 and starts chapter 2. Here's the list of problems, followed by a few answers.

Section 1.9. Exercises 15, 25, 31, 39

Section 2.1. Exercises 13, 18, 23, 28, 29

Section 2.2. Exercises 11, 23, 24, 35

Section 2.3. Exercises 7, 13, 14, 17, 19, 20, 21, 33, 35

Some Answers

2.1-18: The first two columns of AB are Ab_1 and Ab_2. They are equal since b_1 and b_2 are equal.

2.1-28: Since the inner product $u^T v$ is a real number, it equals its transpose. That is, $u^T v = (u^T v)^T = v^T (u^T)^T = v^T u$. Applied here is Theorem 3(d), regarding the transpose of a product of matrices, and Theorem 3(a). The outer product uv^T is an nn matrix. By Theorem 3, $(uv^T)^T = (v^T)^T u^T = vu^T$.

2.2-24: If the equation $Ax = b$ has a solution for each b in $\mathcal{R}^n$, then A has a pivot position in each row, by Theorem 4 in Section 1.4. Since A is square, then the pivots must be on the diagonal of A. It follows that A is row equivalent to the identity matrix I_n . By Theorem 7, matrix A is invertible.

2.3-14: If A is lower triangular with nonzero entries on the diagonal, then these n diagonal entries can be used as pivots to produce zeros below the diagonal. Thus A has n pivots and so it is invertible, by the Invertible Matrix Theorem. If one of the diagonal entries in A is zero, then A will have fewer than n pivots and hence A will be singular.

2.3-20: By the box following the Invertible Matrix Theorem, E and F are invertible and are inverses. So $FE = I = EF$, and therefore matrices E and F commute.